MAST-811: Time Series Analysis (Fall 2021, Andreas Muenchow, Univ. Delaware)
Take-Home-Final-Exam (due Dec. 9, 2021)

In this exercise you are asked to perform a time-domain empirical orthogonal function (EOF) analysis using artificial data u(x,y,t). You will need to set up a correlation matrix, find the eigenvalues $\lambda$ with their corresponding eigenvectors $\phi(x,y)$, and amplitude time series A(t) that decompose the data measurements as

$$u(x,y,t) = \sum_{i=1,N} A_i(t)\, \phi_i(x,y)$$

Assume you sample the scalar pressure field (units of Pascal)

$$u(x,y,t) = Noise(x,y,t) + (a+b*x+c*y)* \begin{cases} \cos(\omega * t) & \text{for } y > 0.5 \\ \cos(10*\omega * t) & \text{for } y < 0.5 \end{cases}$$

at N=9 locations $(x_i, y_j)$ for one year where

$x_i = 0.3*i \quad i=1,2,3$

$y_j = 0.3*j \quad j=1,2,3$

and a time step of $\Delta t$ = 3 hours, i.e., $t=t_k=k*\Delta t$ with k=1,2,3, …, 365*8. Use a=10, b=1, and c=5.

At each location the frequency $\omega=2*\pi/T$ where T=24 hours. Noise(x,y,t) represents Gaussian noise from a random number generator with zero mean, a variance of 1, and a different seed for each location.

1. Find the EOFs for the above field. Map (by hand contouring) the two eigenvectors that explain the largest amount of the variance, plot the corrsponding two amplitude time series, and interpret the results. How much of the total variance is explained by each of the 9 modes?

2. Estimate the auto-spectral density functions for two largest modes and estimate confidence limits for 20 degress of freedom. [Hint: Segment the data into 10 segments and do proper ensemble averaging before you estimate derived properties; recall that this will reduce the frequency resolution from 1/T to 10/T.]

3. Show that mode-1 and mode-2 are indeed orthogonal (uncorrelated) at all frequencies via cross-spectral analysis. Show the confidence level for 20 degrees of freedom and do proper ensemble averaging before you estimate derived properties. [Hint: Use the ensemble averaged fourier transforms of part-2.]

Please ensure that all units are labeled correctly and include a detailed description in your write-up on your interpretation of your results.

MatLab contains extensive matrix routines to solve eigenvalue problems. If you prefer to code in fortran, please access the subroutines via our class web-site, e.g.,

http://muenchow.cms.udel.edu/classes/MAST811/recipes.fortran

TRED2 reduces a real symmetric matrix to a symmetric tridiagonal matrix and
TQLI generates eigenvalues and eigenvectors of the tridiagonal matrix generated above.

You can google these routines; I copied them from “Press et al., 1986: The art of scientific computing”. These codes are the basis of many linear algebra packages such as LINPACK, EISPACK, and, I suspect, even MatLab.