MAST-811: Time Series Analysis (Fall 2021)

Homework Set-1 (due Sept.-23, 2021)

1. Consider the (real) function (sea level x in meters as a function of time t)

$$x(t) = b \cdot \exp(-a\,t) \qquad \text{for } t \geq 0 \text{ } (a = 2 \text{ Hz}, b = 1 \text{ m})$$
$$x(t) = 0 \qquad \text{for } t < 0$$

   a. Calculate the (complex) Fourier Transform X(f) analytically;

   b. Find the absolute value squared Fourier transform $|X(f)|^2 = X(f) \cdot X^*(f)$ where X^* is the complex conjugate of X

   c. Sketch out the distribution of $|X(f)|^2$ with frequency f. What is the maximum value of $|X(f)|^2$ and where would you expect to find it?

   d. Find the frequency where reaches 1/2 of its maximum value;

   e. Can you sample the function x(t) without aliasing? If so, what is the appropriate sampling interval? If not, why?

2. Consider the above function x(t). Assume you have a finite (T=128 seconds) and sampled version of x(t), starting at time t=0. Please start with Δt=1 second and compute the complex absolute value squared discrete Fourier transform $|X(f)|^2$ of the finite, sampled time series x_k. Please plot and label the results. Then do the same for Δt=0.5 seconds and Δt=0.25 seconds. Compare your results with those you derived analytically above for the continueous transform X(f). Comment and discuss on similiarities and discrepancies. [Please provide source codes.]

Please explain your reasoning, interpretations, and problems encountered, attach your code, and properly label all plots with units.