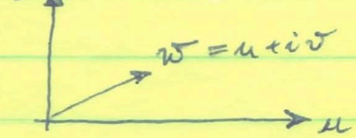
EOF on vectors



$$w(t, x_m) = u(t, x_m) + i\,v(t, x_m)$$

$m = 1, 2, \dots M$

hodograph $\qquad$ w complex

set-up correlation matrix

$$R_{pq} = \langle w_p w_q^* \rangle = \langle (u_p + i v_p)(u_q - i v_q) \rangle$$
$$= \langle u_p u_q + i(v_p u_q - u_p v_q) + v_p v_q \rangle$$
$$R_{qp} = \langle w_q w_p^* \rangle = \langle u_q u_p + i(v_q u_p - u_q v_p) + v_p v_q \rangle$$

$\therefore \; R_{pq} = R_{qp}^*$ $\qquad$ i.e., the complex correlation matrix is Hermetian

→ eigenvalues are all real

set $\underset{\sim}{R} = \underset{\sim}{A} + i\underset{\sim}{B}$ $\qquad$ the the eigenvalue problem becomes

$$(\underset{\sim}{A} + i\underset{\sim}{B})(\phi + i\psi) = \lambda(\phi + i\psi)$$

$\vec{u} = (u_1, u_2, \dots, u_M)$
$\vec{v} = (v_1, v_2, \dots v_M)$

or $$\begin{pmatrix} \underset{\sim}{A} & -\underset{\sim}{B} \\ \underset{\sim}{B} & \underset{\sim}{A} \end{pmatrix} \begin{pmatrix} \phi \\ \psi \end{pmatrix} = \lambda \begin{pmatrix} \phi \\ \psi \end{pmatrix}$$

$\begin{pmatrix} -\psi \\ +\phi \end{pmatrix}$ is also an eigenvector

→ the "complex" or vector version of an EOF is a straightforward extension of the ordinary or scalar EOF

. . . . start class # 22

Significance and Errors in EOFs

Test 1 : Can a specific mode be distinguished from random noise?

Overland and Preisendorfer (1982). Monthly Weather Review
→ Monte Carlo technique of hypothesis testing

- use artificial data from a random number generator to "simulate" your real data
- perform an EOF on, say, 100 random data sets
- order the resulting, say, 100 estimates of the eigenvalue for each mode
- test if your estimate of the eigenvalue from mode m of your real data is larger than ~~the~~ 95% of the eigenvalues ~~of~~ from mode m from the random data
- if it is, interpret the result as a rejection of the Null hypothesis that your data is not significantly different from random data at 95% confidence

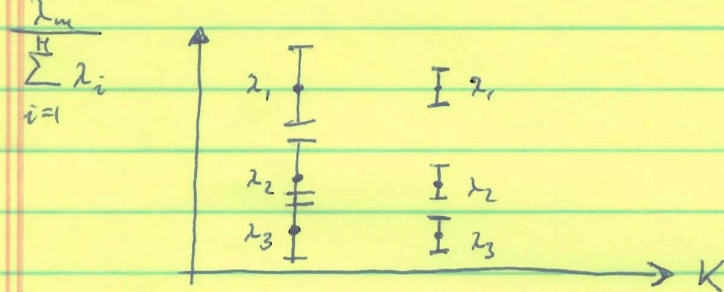
K / M		20	100	1000	# of data in time series (# of ensemble, realisations)
9	j=1	29.8	18.3	13.1	mode 1 out of 9
	j=2	22.0	15.7	12.4	mode 2 out of 9
	j=5	11.6	11.6	11.3	mode 5 out of 9
36	j=1	15.0	6.9	3.9	mode 1 out of 36
	j=2	12.7	6.5	3.8	mode 2 out of 36
	j=5	8.7	5.2	3.5	mode 5 out of 36
100	j=1	10.5	4.0	1.7	
	j=2	9.3	3.7	1.7	
	j=5	7.4	3.2	1.6	

Test 2: What are approximate sampling errors of an EOF?

North et al. (1982). Monthly Weather Review
→ linear error/perturbation analysis

$\delta \lambda_m \approx \lambda_m \sqrt{2/K}$ standard error in eigenvalue

$\delta \phi_m \approx \dfrac{\delta \lambda_m}{\lambda_m - \lambda_n} \phi_n$ standard error in eigenvector



degeneracy of the system → two "modes" have the "same" eigenvalue
→ any linear combination of the two modes is also an eigenvector

to find eigenvalues and eigenvectors use
Numerical Recipes routines

TRED2
TQLI

very efficient and accurate
→ the same cannot be said for SVDCMP → singular value decomposition