Wavelets and Signal Processing

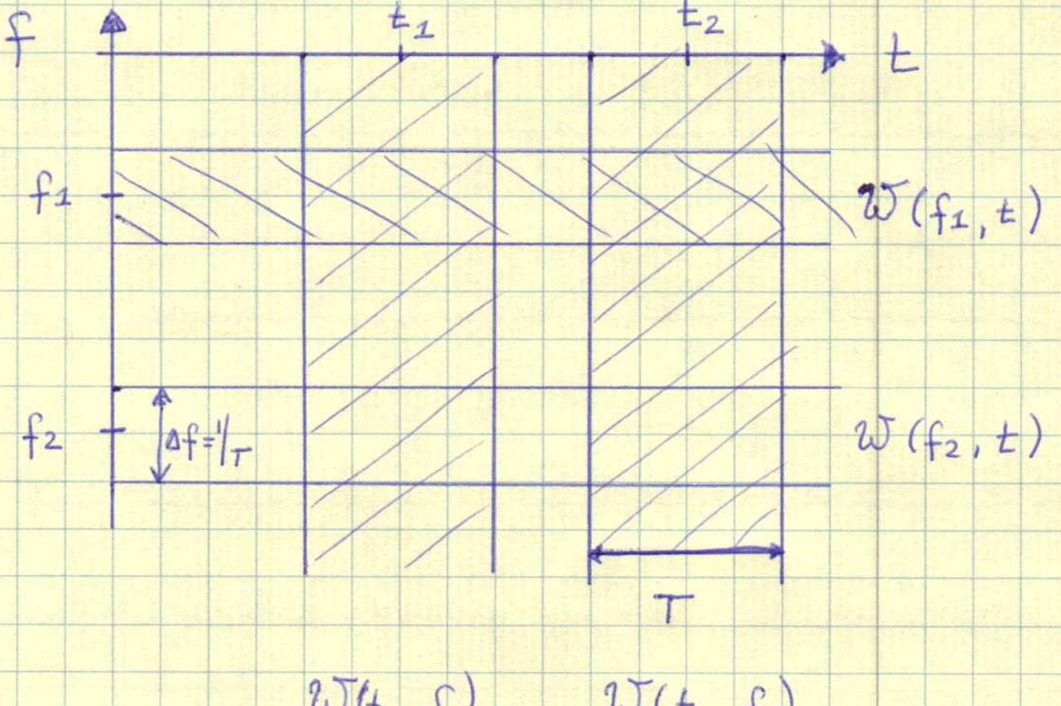
As an alternative to Short-Time Fourier Transform

or Windowed Fourier Transform Methods

$$W_{xx}(f,t) = \int R_{xx}(\tau,t)\, e^{-j 2\pi f \tau}\, d\tau$$

which is the Fourier Transform of $R_{xx}(\tau,t)$ performed on a sliding segment or window of length T. The frequencies f then are a sequence $\frac{1}{T}, \frac{2}{T}, \frac{3}{T}, \ldots, 1/2\Delta t = \frac{N/2}{T}$

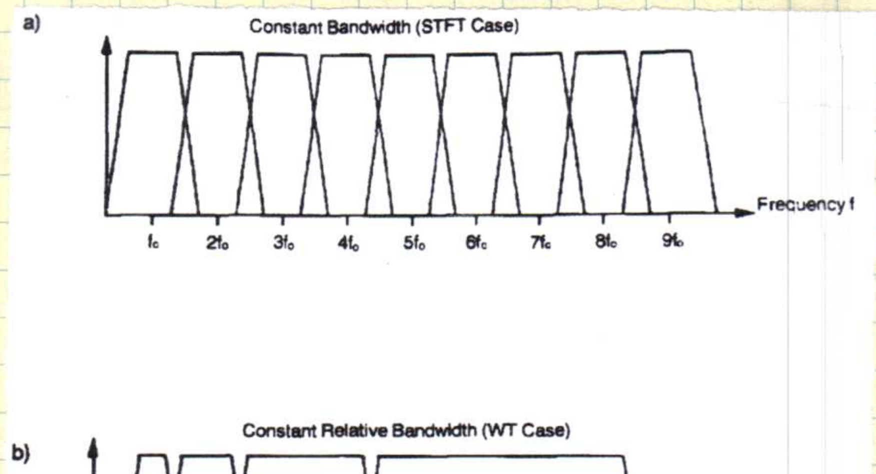
at each time t with $T = N \cdot \Delta t$



Note that both the time and frequency axes consist~~s~~ of discrete set of windows each of length (time-resolution) T and (frequency resolution) $1/T$ $\rightarrow \Delta f \cdot \Delta T = 1$

The time-frequency localization is said to depend on the scale T that in Fourier Analyses must be chosen a priori. We always for each short window or data segment have $N/2$ discrete frequencies at which we evaluate our data.

The wavelet analysis removes this stringent scale-T dependence as it replaces the fixed bandwidth $\Delta f = 1/T = \text{const.}$ with a bandwidth $\Delta f = \Delta f\,(f)$ that varies with frequency:



Short Fourier / Transform

$\Delta f = \text{const.}$

Wavelet Transform

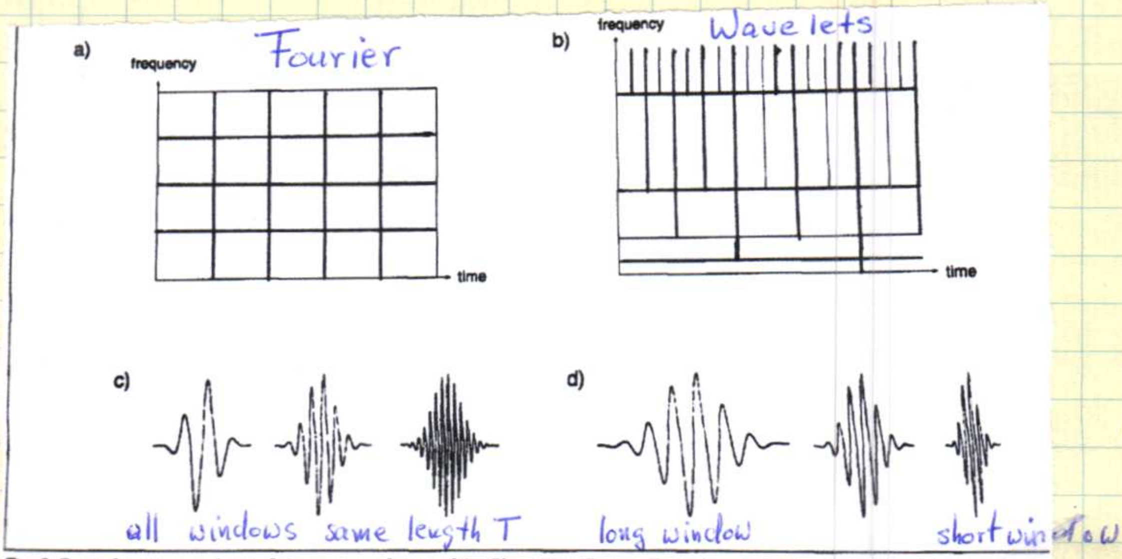
$\Delta f = \Delta f\,(f)$

narrow band — wide band, short T

long T — short T

for long periods — for short periods

So the time-frequency or time-scale domain looks like



Resolution in (t, f) plane

Base functions at

Fig. 2. Basis functions and time-frequency resolution of the Short-Time Fourier Transform (STFT) and the Wavelet Transform (WT). The tiles represent the essential concentration in the time-frequency plane of a given basis function. (a) Coverage of the time-frequency plane for the STFT, (b) for the WT. (c) Corresponding basis functions for the STFT, (d) for the WT ("wavelets").

Fourier $\quad \Delta f = \text{const}$

Wavelet $\quad \Delta f = \text{const} \cdot f \qquad$ or $\dfrac{\Delta f}{f} = \text{const}$

The Continuous Wavelet Transform (CWT) conserves

$$\Delta f(f) \cdot T(f) = 1$$

but unlike the Fourier Transform both Δf and T depend on frequency. This is accomplished with a scaled (stretched or compressed) version of the same prototype wavelet $\psi(t)$, e.g.,

$$\psi_a(t) = \frac{1}{\sqrt{|a|}}\, \psi(t/a)$$

where a is the scale factor