MAST-811: Time Series Analysis (Fall 2022)

Project-2 (due Oct.-13, 2022)

1. Consider the exponential decay x(t) of a radioactive material discharged into the ocean measured in ppb (parts per billion) as a function of time t in years. The time constant or exponential decay rate is a=1/year

$$x(t) = \exp(-a\,t) \qquad \text{for } t>0\ (a>0)$$
$$x(t) = 0 \qquad \text{for } t<0$$

   a. [20 pts] Find the ***complex*** Fourier Transform ***X(f)*** and its ***real*** absolute value square $|X(f)|^2$;

   b. [10 pts] Plot the function $|X(f)|^2$ as a function of frequency f. What is the maximum value of $|X(f)|^2$ and where would you expect to find it?

   c. [5 pts] Find the frequency where reaches 1/2 of its maximum value;

   d. [5 pts] Can you sample the function x(t) without aliasing? If so, what is the appropriate sampling interval? If not, why?

2. [40 pts] Consider the above function x(t). Assume you have a finite (T=64 year) and sampled version of x(t), starting at time t=0 or 1958 at the height of the Cold War. Please start with Δt=1 year (annual expedition) and compute $|X(f)|^2$ from the ***complex discrete Fourier transform X(f)*** of the finite, sampled time series $x(t_k=k*\Delta t)$. Please plot and label the results. Then do the same for Δt=0.5 years (2 expeditions each year) and Δt=0.25 years (4 expeditions each year). Compare your results with those you derived analytically above for the continueous transform X(f). Comment and discuss on similiarities and discrepancies of analytical (exact) and sampled (approximate) versions of the signal in full sentences that reflect on what you may have learnt in-class up to this point. [Please provide source codes, too, as always.]

3. Suppose that you are capable of taking infinite samples from a physical system that contains energy in the frequency range 0 to 1 Hz. You also know that somehow high frequency noise at 60 Hz was introduced into your system.

   a. [10 pts] Assuming you are only interested in studying the true physical process (from 0 to 1 Hz), what sampling rate would you use? Why?

   b. [10 pts] Suppose you cannot sample faster than 5 Hz, do you expect aliasing to occur within the frequency range 0 to 1 Hz? Why? Is there anything you can do?