MAST-811: Time Series Analysis (Fall 2015)

Computer Exercise-3 (due Nov.-17, 2015)

Use a pseudo-random number generator to generate a single sequence of 60,000 numbers as

$$x(t) = N(\mu_x=0,\sigma_x^2=1) + a_1*\cos(w_1*t)+a_2*\cos(w_2*t)+a_3*\cos(w_3*t)$$

where $N(\mu_x=0,\sigma_x^2=1)$ a Gaussian normal distribution that has zero mean and variance of 1 and $w_i=2*\pi/T_i$ with $T_i=\{60, 6, 12.05\}$ months and $a_i=\{0.2,0.5,2\}$ meters for $i=\{1,2,3\}$. You are asked to sample this time series at $\Delta t=1$ month. Consider these data to represent random sea level oscillations in the sub-tropical Atlantic Ocean in meters sampled with $\Delta t=1$ month.

1. Compute and graph the time series and estimates of the autopower spectra

   $$Sxx(f)=2/T\ |X(f)|^2.$$

   Provide 95% confidence limits for each of your estimates.

2. Write a program to implement a box car filter and use this filter to extract the 5-year signal as a time series y(t). As in (1) above, compute and graph the time series and estimates of the auto-spectra

   $$Syy(f) = 2/T\ |Y(f)|^2$$

3. Write a program to estimate the complex, linear transfer function H(f) between the raw data x(t) and the filtered data y(t). Recall that the complex function H(f) can be written as

   $$H(f) = |H(f)| * \exp[-j\ \theta(f)]$$

   where |H)f)| represents the gain and $\theta(f)$ the phase of the transfer function and

   $$Y(f) = H(f) * X(f).$$

Please describe to me what you have done, how you have done it, and what you have learnt from these exercises. Please attach your programs to your write-up, and properly label all plots. The plots do not have to be "fancy," but make sure that all axes are labeled properly and that the units are correct (by hand if necessary). Please make sure to mention properties related to sampling rate and record length in both time and frequency domains.