MAST-811: Time Series Analysis (Fall 2022, Andreas Muenchow, Univ. Delaware)
Take-Home-Final-Exam (due Dec. 14, 2022)

In this exercise you are asked to perform a time-domain empirical orthogonal function (EOF) analysis using artificial data u(x,y,t). You will need to set up a correlation matrix, find the eigenvalues λ with their corresponding eigenvectors $\phi(x,y)$, and amplitude time series A(t) that decompose the data measurements as

$$u(x,y,t) = \sum_{i=1,N} A_i(t)\ \phi_i(x,y)$$

Assume you sample the scalar field

$$u(x,y,t) = \text{Noise}(x,y,t) + (a+b*x+c*y)* \begin{cases} \cos(\omega*t) & \text{for } y > 0.5 \\ \cos(2*\omega*t) & \text{for } y < 0.5 \end{cases}$$

For $\omega=2*\pi/T$ with T=24 hours at N=9 locations (x_i,y_j) where

$$x_i = 0.3*k \qquad k=1,2,3$$
$$y_j = 0.3*l \qquad l=1,2,3$$

and a time step of Δt = 1 hour, i.e., $t=t_r=r*\Delta t$ r=1,2,3, …, 1000

Noise(x,y,t) represents Gaussian noise from a random number generator with zero mean, a variance of 1, and a different seed for each location.

1. Find the EOFs for the above field. Map (by hand contouring) the eigenvectors, plot the amplitude time series, and interpret the results. How much of the total variance is explained by each of the 9 modes?
2. Show that the first 2 modes are indeed orthogonal by demonstrating that the timeseries $A_1(t)$ and $A_2(t)$ are uncorrelated at all frequencies.
3. Repeat the above, but set the noise term to zero.

MatLab contains extensive matrix routines to solve eigenvalue problems. If you prefer to code in fortran, please access the subroutines via our class web-site, e.g.,

http://muenchow.cms.udel.edu/classes/MAST811/recipes.fortran

TRED2 reduces a real symmetric matrix to a symmetric tridiagonal matrix and
TQLI generates eigenvalues and eigenvectors of the tridiagonal matrix generated above.

You can google these routines; I copied them from Press et al., 1986: The Art of Scientific Computing. These codes are the basis of many linear algebra packages such as LINPACK, EISPACK, and, I suspect, even MatLab.

Please include a detailed description in your write-up on your interpretation and reflections of your results.