MAST-811: Time Series Analysis (Fall 2022)

Homework-1 (due Sept.-15, 2022)

Please familiarize yourself again with complex numbers and algebra that we will use throughout this course. Recall that a complex number or time series z(t) has a real part x(t) and an imaginary part y(t) in the sense that $z(t) = x(t) + j\, y(t)$ where $j=\sqrt{(-1)}$. Note that the complex conjugate $z^*(t) = x(t) - j\, y(t)$

1. [2.5 pts] Proof via simple algebra that the product of z(t) and z*(t) is real, that is, its imaginary part is zero. Show all your work
2. [2.5 pts] What is the real part of z(t)/z*(t)? Show all your work.
3. [2.5 pts] What is the imaginary part of z(t)/z*(t)? Show all your work.

The Fourier series of a real function

$$X1(t) = \sum_i a_i \cos(2\pi/T\ t) + b_i \sin(2\pi/T\ t)$$

with "i" an integer index $i=0,1,2,\ldots,\infty$ with $X1(t)=X1(t+T)$. The same real function X1(t) can also be written (more concisely) in complex notation as

$$X2(t) = \sum_i c_i \exp(j\ 2\pi/T\ t)$$

with "i" an integer index $i=-\infty,\ldots,-2,-1,0,1,2,\ldots,\infty$ with $c_i = a_i - j\, b_i$ and $c_{-i} = a_i + j\, b_i$ Note that c_i and c_{-i} are complex conjugate, that is, $c_i=c_{-1}^*$

4. [2.5 pts] How does the product c_i times c_{-i} relate to the real amplitudes a_i and b_i?

5. [10 pts] Show algebraically that

$$X1(t) = X2(t)$$

Recall Euler's formula, that is, $\exp(j\theta) = \cos(\theta) + j \sin(\theta)$ where the phase angle $\theta=\theta(t)$ can be any real function of time, say, including $\theta(t)=2\pi/T\ t$. Recall also that the sum $\sum_i c_i$ with i in the interval $[-\infty,\infty]$ can be broken into three different sums

$$\sum_i c_i = \sum_k c_k + c0 + \sum_m c_m$$

where k is in the interval $[-\infty,-1]$ and m is in the interval $[1, \infty]$. Please showyou're your work and all algebraic steps in all your derivations.

You may find that

https://www.princeton.edu/~wbialek/intsci_web/dynamics2.2.pdf

provides useful pointers to refresh your familiarity with complex numbers, notation, and algebra as do any engineering math/algebra books that you may have encountered during your prior education.