MAST-811: Time Series Analysis (Fall 2016)

Homework Set-2 (due Oct.-4, 2016)

1. Consider the function

$$x(t) = \exp(-a\,t) \qquad \text{for } t>0 \ (a>0)$$
$$x(t) = 0 \qquad \text{for } t<0$$

   a. Find the ***complex absolute value squared Fourier transform*** $|X(f)|^2$;

   b. Sketch out the distribution of $|X(f)|^2$ with frequency f. What is the maximum value of $|X(f)|^2$ and where would you expect to find it?

   c. Find the frequency where reaches 1/2 of its maximum value;

   d. Can you sample the function x(t) without aliasing? If so, what is the appropriate sampling interval? If not, why?

2. Consider the above function x(t). Assume you have a finite (T=512 seconds) and sampled version of x(t), starting at time t=0. Please start with $\Delta t=1$ second and compute the ***complex absolute value squared discrete Fourier transform*** $|X(f)|^2$ of the finite, sampled time series x_k. Please plot and label the results. Then do the same for $\Delta t=0.5$ seconds and $\Delta t=0.25$ seconds. Compare your results with those you derived analytically above for the continueous transform X(f). Comment and discuss on similiarities and discrepancies. [Please provide source codes.]

3. Suppose that you are capable of taking infinite samples from a physical system that contains energy in the frequency range 0 to 1 Hz. You also know that somehow high frequency noise at 60 Hz was introduced into your system.

   a. Assuming you are only interested in studying the true physical process (from 0 to 1 Hz), what sampling rate would you use? Why?

   b. Suppose you cannot sample faster than 5 Hz, do you expect aliasing to occur within the frequency range 0 to 1 Hz? Why? Is there anything you can do?