

1. Estimate the 50% confidence limits of an auto-spectral estimate  $P_n$  for 2 degrees of freedom, that is, use  $\chi^2_2$  ;
2. Estimate 99% confidence limits of an auto-spectral estimate  $P_n$  for 20 degrees of freedom, that is, use  $\chi^2_{20}$  ;
3. Estimate 95% confidence limits of an auto-spectral estimate  $P_n$  for 40 degrees of freedom, that is, use  $\chi^2_{40}$  ;
4. Estimate 95% confidence limits of an estimate  $P_n$  for 40 degrees of freedom, but instead of  $\chi^2_{40}$  use the Gaussian probability density function or normal distribution  $N(0,1)$  instead. [Recall the Central Limit Theorem which states that any sum of independent random variables with arbitrary probability density distributions converges towards the Gaussian Normal distribution as the number of summands (degrees of freedom) increases towards infinity.]

Please explain your reasoning, interpretations, and problems encountered.

You may find the following two web-sites useful to find values for the probability of a random variable to fall above a certain value:

Gaussian Normal Distribution  $N(\mu,\sigma)$ :

<https://homepage.stat.uiowa.edu/~mbognar/applets/normal.html>

Chi-Square ( $\chi^2$ ) Distribution:

<https://homepage.stat.uiowa.edu/~mbognar/applets/chisq.html>