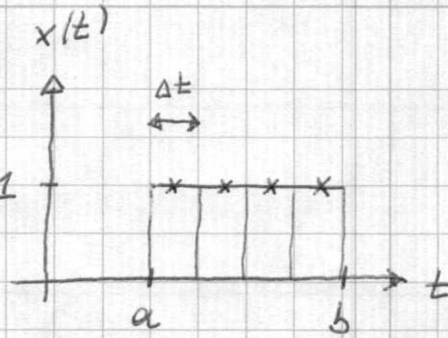
Sept.-15, 2022

$$\int_a^b x(t)\,dt$$

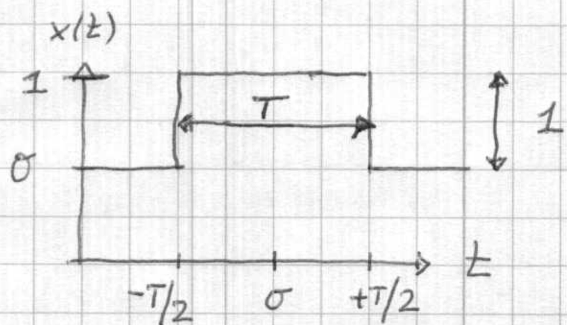


$$\lim_{\Delta t \to 0} \sum_{i=1}^{N} x(t_i)\cdot\Delta t = \int_a^b x(t)\,dt$$

$$\sum_{k=1}^{N} X(t_k) = \sum_{k=1}^{N} 1$$

for $x(t_k) = 1$

$$x(t) = \begin{cases} 1 & t \in [\overset{-T/2}{\cancel{0}}, T/2] \\ 0 & |t| > T/2 \end{cases}$$



$$\int_{-\infty}^{+\infty} x(t)\,dt = \int_{-\infty}^{+T/2} x(t)\,dt + \int_{-T/2}^{+T/2} x(t)\,dt + \int_{+T/2}^{\infty} x(t)\,dt$$

$$= \int_{-\infty}^{+T/2} 0\,dt + \int_{-T/2}^{+T/2} 1\,dt + \int_{+T/2}^{\infty} 0\,dt$$

$$= 0 + t\Big|_{-T/2}^{+T/2} + 0$$

$$= 0 + \frac{T}{2} - (-)\frac{T}{2} + 0$$

$$= T$$

$$s(t) = \begin{cases} 1 & |t| < T/2 \\ 0 & |t| > T/2 \end{cases}$$

$$S(f) = \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi f t}\, dt = T \cdot \frac{\sin \pi f T}{\pi f T}$$

Think of this as the Fourier Series coefficients $c_n = C(f_n)$

What is $\int_{-\infty}^{+\infty} S(f)\, df$? $\sum_{k=1}^{K} S(f_k) \cdot \Delta f$

$$\int_{-\infty}^{+\infty} T \frac{\sin(\pi f T)}{\pi f T}\, df \underset{\sigma = \pi f T}{=} T \int_{-\infty}^{+\infty} \frac{\sin(\sigma)}{\sigma} \cdot \underbrace{\frac{1}{\pi T}\, d\sigma}_{df}$$

change of variables

$$\frac{d\sigma}{df} = \pi T \quad \Rightarrow d\sigma = \pi \cdot T \cdot df$$
$$\Rightarrow df = d\sigma / \pi \cdot T$$

$$= \frac{\cancel{T}}{\pi \cancel{T}} \int_{-\infty}^{+\infty} \frac{\sin(\sigma)}{\sigma}\, d\sigma = \frac{\cancel{T}}{\cancel{\pi} \cancel{T}} \cdot \cancel{\pi} = 1$$

Note that this result does not depend on T !

BUT

$$\lim_{T \to \infty} S(f) = \lim_{T \to \infty} T \cdot \frac{\sin(\pi f T)}{\pi f T} \qquad \text{with } \Delta f = 1/T$$

$$\lim_{\Delta f \to 0} S(f) = \lim_{\Delta f \to 0} \frac{1}{\Delta f} \cdot \sin \pi n \cdot \Delta f / \Delta$$

Sept. - 15, 2022

Convolution Integral

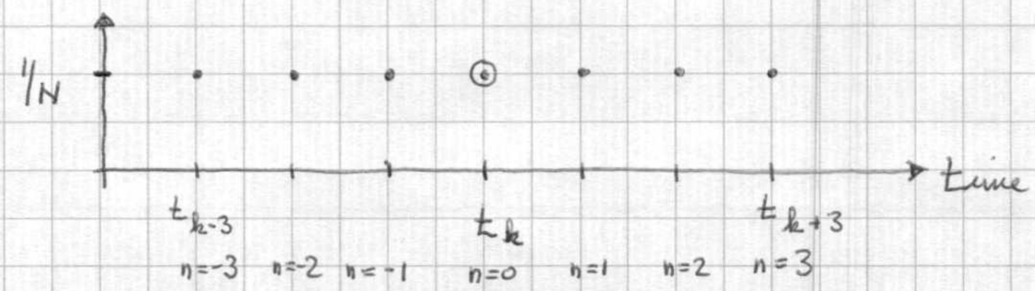
$$y(t) = \int_{-\infty}^{+\infty} h(\tau) \cdot x(t-\tau) \, d\tau \equiv \underbrace{\underset{\text{Filter Weights}}{h} * \underset{\text{Data}}{x}}_{\text{Filtered Data}}$$

Def. of Convolution

$$\approx \sum_{i=-n}^{i=+n} h(\tau_i) \cdot x(t-\tau_i) \; \Delta\tau$$

Example

$$h(\tau_i) = \frac{1}{N} \qquad N = 2 \cdot n + 1$$



So

$$y(t_k) = \frac{1}{N} \Big[x(t_{k-3}) + x(t_{k-2}) + x(t_{k-1}) + x(t_k) + x(t_{k+1}) + x(t_{k+2}) + x(t_{k+3}) \Big]$$

Simple Running Average (Box Car)

Or generally

$$y(t_k) = h_{-3} \cdot x(t_{k-3}) + h_{-2} \, x(t_{k-2}) + h_{-1} \, x(t_{k-1}) + h_0 \, x(t_k) + h_1 \, x(t_{k+1}) + h_2 \, x(t_{k+2}) + h_3 \, x(t_{k+1})$$

for arbitrary $h = h(\tau)$

fancy Running Average