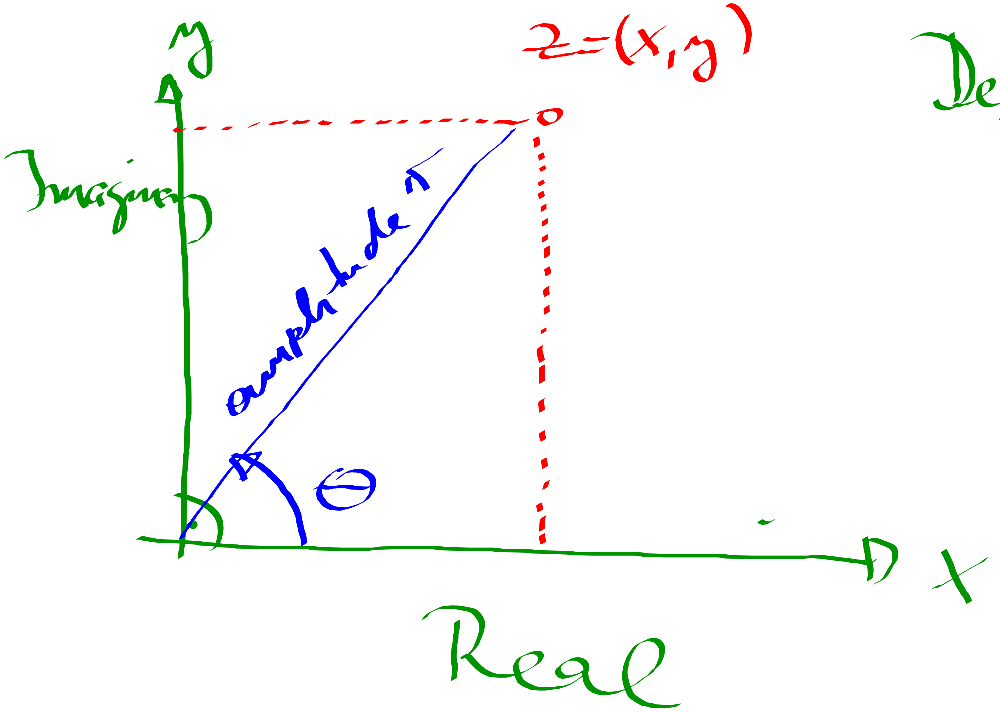


Define a complex unit vector

$$j \equiv \sqrt{-1}$$

$$y = r \sin \theta$$

$$x = r \cos \theta$$

Complex Exponentials

$$e^{j\theta} = \cos \theta + j \sin \theta$$

Fourier Series

Joseph Fourier 1768-1830 French Physicist

In 1807 boldly pronounced

ANY PERIODIC FUNCTION

$$\underset{\text{Data}}{x}(t) = x(t+T)$$

can be represented

$$\underset{\text{Data}}{x}(t) = \sum_{i=0}^{\infty} a_i \cos\left(\frac{2\pi}{T} i t\right) + b_i \sin\left(\frac{2\pi}{T} i t\right)$$

$$\int_{-T/2}^{+T/2} \cos\left(m \frac{2\pi}{T} t\right) \cdot \cos\left(n \frac{2\pi}{T} t\right) = \begin{cases} 0 & n \neq m \\ T/2 & n = m \geq 1 \\ T & n = m = 0 \end{cases}$$

$$\int_{-T/2}^{+T/2} \sin\left(m \frac{2\pi}{T} t\right) \cdot \sin\left(n \frac{2\pi}{T} t\right) dt = \begin{cases} 0 & m \neq n \\ T/2 & n = m \end{cases}$$

$$\int_{-T/2}^{+T/2} \sin\left(m \frac{2\pi}{T} t\right) \cdot \cos\left(n \cdot \frac{2\pi}{T} t\right) dt = 0$$

$$\int_{-T/2}^{+T/2} X(t) \cdot \cos\left(n \frac{2\pi}{T} t\right) dt$$

Data

is expanded

on this

Fourier

series

$$\cos\left(n \frac{2\pi}{T} t\right) dt$$

$$= \int_{-T/2}^{+T/2} \sum_{m=0}^{\infty} a_m \cos\left(\frac{2\pi}{T} m t\right) + b_m \sin(\ldots)$$

$n=5$

$$= \sum_{m=0}^{\infty} a_m \int \cos\left(\frac{2\pi}{T} m t\right) \cdot \cos\left(n \frac{2\pi}{T} t\right)$$

$$+ b_m \int \sin\left(\frac{2\pi}{T} m t\right) \cdot \cos\left(n \frac{2\pi}{T} t\right) \to 0$$

$$= a_n \cdot T/2 \qquad n = m$$

frequency $f_n = \frac{2\pi}{T} \cdot n$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} \boxed{X(t)} \cdot \cos \boxed{\frac{2\pi}{T} \cdot n} t \, dt$$

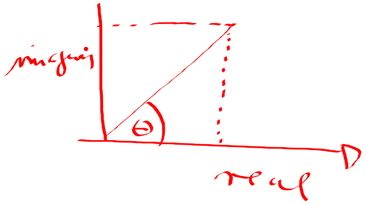
$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} X(t) \cdot \sin\left(\frac{2\pi}{T} n t\right) dt$$

$$x(t) = \sum_{n=0}^{\infty} a_n \cos(\ldots) + b_n \cdot \sin(\ldots)$$

$$\theta = (\ldots) = \frac{2\pi}{T} n t$$

$$= \sum_{n=-\infty}^{+\infty} c_n \, e^{\underbrace{j \frac{2\pi}{T} \cdot n t}_{\theta}}$$

$$e^{j\theta} = \cos\theta + j \sin\theta$$



$$\sin\theta = \frac{1}{2j}\left(e^{+j\theta} - e^{-j\theta}\right)$$

$$\cos\theta = \frac{1}{2}\left(e^{+j\theta} + e^{-j\theta}\right)$$

$$\frac{1}{j} = \frac{j}{j \cdot j} = \frac{j}{-1} = -j$$

What about non-periodic functions

$T \longrightarrow \infty$

$\frac{1}{T} = \Delta f$ frequency step $\frac{n}{T} = f = n \cdot \Delta f$

f for frequency

$$\underset{\text{Data}}{x(t)} \equiv \lim_{\substack{T \to \infty \\ \Delta f \to df \\ n \cdot \Delta f \to f}}$$