$$S(f) = \int_{-\infty}^{+\infty} s(t)\, e^{-j 2\pi f t}\, dt \equiv \mathcal{F}(s(t))$$

$$s(t) = \int_{-\infty}^{+\infty} S(f)\, e^{+j 2\pi f t}\, df \equiv \mathcal{F}^{-1}(S(f))$$

$$s(t) = \underbrace{\underbrace{x(t) \cdot \Pi(t,T)}_{\text{continuous Data in finite intervall}} \cdot \sum_{n=1}^{N} \delta(t - n \cdot \Delta t)}_{\text{discrete (digital) Data in finite interval } T}$$

continuous process — Data Window — Sampling Scheme

$x(t)$

$$\Pi(t,T) = \begin{cases} 1 & |t| \le T/2 \\ 0 & |t| > T/2 \end{cases}$$

$$\delta(t - t_0) = \begin{cases} \infty & t = t_0 \\ 0 & t \neq t_0 \end{cases} \quad \text{with} \int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$

$$S(f) = \int_{-\infty}^{+\infty} \Pi(t,T)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-T/2}^{+T/2} 1 \cdot e^{-j2\pi f t}\, dt$$

$$= \frac{-1}{j2\pi f \cancel{t}} \left[ e^{-j2\pi f t} \right]_{t=-T/2}^{t=+T/2}$$

$$= \frac{-1}{j2\pi f} \left[ e^{-j2\pi f \cdot T/2} - e^{-j2\pi f(-T/2)} \right]$$

$$= \frac{+1 \cdot T}{j2\pi f \cdot T} \left[ e^{+j\cancel{2}\pi f T/\cancel{2}} - e^{-j\cancel{2}\pi f \cdot T/\cancel{2}} \right]$$

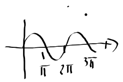
$$e^{+j\pi f T} - e^{-j\pi f T}$$

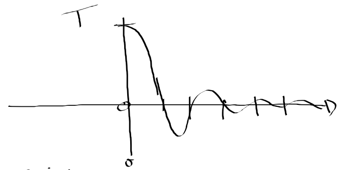
$$\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

$$= T \frac{\sin(\pi f T)}{\pi f T}$$

$$\frac{\sin x}{x} = \mathrm{sinc}(x)$$

$$= T\, \mathrm{sinc}(\pi f T)$$




$fT = 0, 1, 2, 3, 4$ $\pi$

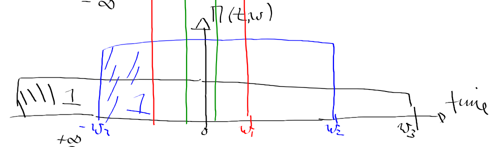
$$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{\cos x}{1} = \cos 0 = 1$$

L'Hospital

Dirac's Delta function

$$\Pi(t,w) = \begin{cases} 1/w & -\frac{w}{2} \le t \le +w/2 \\ 0 & |t| > w/2 \end{cases}$$

$$\text{area} = \int_{-\infty}^{+\infty} \Pi(t,w)\,dt = 1$$




$$\lim_{w\to 0}\int_{-\infty}^{+\infty} \boxed{\Pi(t,w)}\, x(t)\,dt =$$

$$= \lim_{w\to 0} \frac{1}{w}\underbrace{\int_{-w/2}^{+w/2} x(t)\,dt}$$

$$= \lim_{w\to 0} \cancel{\frac{1}{w}}\; x(w=\xi)\cdot\cancel{w} \qquad \xi \in \left[-\frac{w}{2}, +\frac{w}{2}\right]$$

$$= x(0)$$

Define this sequence as

$$\boxed{\int_{-\infty}^{+\infty} \delta(t-0)\,x(t)\,dt = x(t=0) \quad \text{sifting operation}}$$

$$\int_{-\infty}^{+\infty} \delta(t-t_0)\,x(t)\,dt = x(t_0)$$

Informal, sloppy way

$$\delta(t) = \begin{cases} \infty & t=0 \\ 0 & t\neq 0 \end{cases}$$

with

$$\int_{-\infty}^{+\infty} \delta(t)\,dt = 1$$

$$\Pi(t,T) = \begin{cases} 1 & |t| \le T/2 \\ 0 & |t| \ge T/2 \end{cases} = s(t)$$

$$S(f) = \mathcal{F}(\Pi(t,T))$$


$$S(f) = T \cdot \mathrm{sinc}(fT)$$

As $T \to \infty$ for $\Pi(t,T)$

$$S(f) \longrightarrow \delta(f)$$

$$\cos\theta = \frac{1}{2}\left(e^{+j\theta} - e^{-j\theta}\right)$$

$$s(t) = A \cos 2\pi f_0 t$$

$$S(f) = \mathcal{F}(s(t)) = \int_{-\infty}^{+\infty} A \cos(2\pi f_0 t) \cdot e^{-j 2\pi f t} \, dt$$

$$= \frac{A}{2} \int 1 \cdot \underbrace{e^{-j 2\pi \overbrace{(f-f_0)}^{f'} t}} + 1 \cdot e^{+j 2\pi \overbrace{(f+f_0)}^{f'} t} \, dt$$

$$S(f) = \frac{A}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right]$$