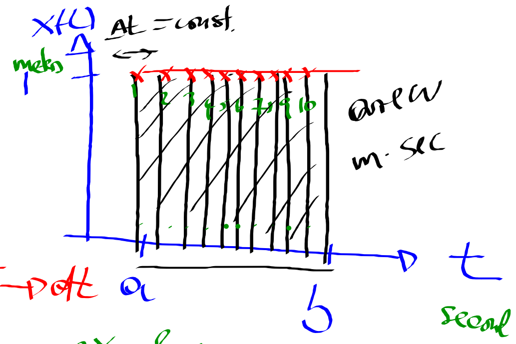
$$\int_{t=a}^{t=b} \underbrace{x(t)}_{\text{meters}}\, \underbrace{dt}_{\text{seconds}}$$



$\Delta t \to 0 \to dt$ small

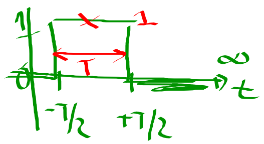
example

$$\sum_{k=1}^{N=10} x(t_k)\, \Delta t = \sum_{k=1}^{N} 1 \cdot \Delta t$$

$$= \Delta t + \Delta t + \Delta t + \Delta t + \Delta t + \Delta t + \Delta t + \Delta t + \Delta t + \Delta t$$

$$= N \cdot \Delta t$$ in meters · seconds

$$\int_{-\infty}^{+\infty} x(t) \cdot dt = \int_{-\infty}^{-T/2} \underbrace{x(t)}_{0} dt \qquad 0$$



$$+ \int_{-T/2}^{+T/2} \underbrace{x(t)}_{1} dt \qquad + T$$

$$x(t) = \begin{cases} 1 & |t| < T/2 \\ 0 & \text{otherwise} \end{cases} \quad + \int_{+T/2}^{\infty} \underbrace{x(t)}_{0} dt \qquad + 0$$

$$\Pi(t,T) = \begin{cases} 1 & |t| < T/2 \\ 0 & |t| > T/2 \end{cases}$$

$$\mathcal{F}(\Pi(t,T)) = \int_{-\infty}^{+\infty} \ldots\, dt = \boxed{T\,\frac{\sin \pi f T}{\pi f T}} = S(f)$$

think of these as the Fourier Series coefficents $C_n = C(f_n) \approx S(f_k)$

What is $\int_{-\infty}^{+\infty} S(f)\, df$

$$\approx \cancel{\sum_{k=1}^{\infty} S(f_k)\cdot \Delta f}$$

$$\int_{-\infty}^{+\infty} S(f)\,df = \int_{f=-\infty}^{f=+\infty} T\cdot \frac{\sin \pi f T}{\pi f T}\, df$$

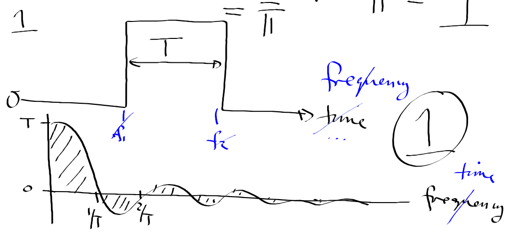
$$\left[\int_{-\infty}^{+\infty} \frac{\sin\sigma}{\sigma}\, d\sigma = \pi\right.$$

$\sigma = \pi \cdot f \cdot T = \sigma(f)$

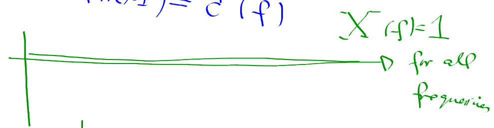
$\frac{d\sigma}{df} = \pi T \quad \Downarrow \quad df = \frac{d\sigma}{\pi T}$

$$\int_{-\infty=\sigma}^{+\infty=\sigma} \cancel{T}\, \frac{\sin\sigma}{\sigma}\, \frac{d\sigma}{\pi \cdot \cancel{T}} = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{\sin\sigma}{\sigma}\, d\sigma$$

$$= \frac{1}{\pi} \cdot \pi = 1$$



$\mathcal{F}(x(t)=1) = \delta(f)$



$\mathcal{F}^{-1}(X(f)) = \delta(t)$

Convolution Integral

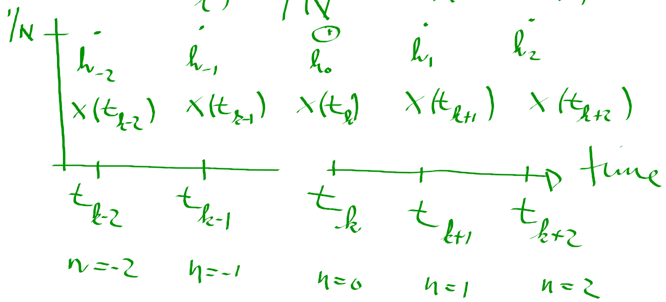
$$y(\underline{t}) = \int_{-\infty}^{+\infty} h(\tau)\cdot x(t-\tau)\,d\tau$$

new data (filtered data) filter weight Data

$$\underline{\underline{\text{Define}}} \quad h * x$$

$$y(\underline{t}) \approx \sum_{i=-n}^{i=+n} h(\tau_i)\cdot x(\underline{t}-\tau_i)\,\Delta\tau$$

Ex.: $h(\tau_i) = 1/N \qquad N = 2\cdot n+1$



$$y(t) = \underbrace{\frac{1}{N}}_{h_i} \sum_{i=-n}^{+n} x(t_{k+n})$$

Simple running average

$$h = \underline{\underline{h(\tau_i)}} =$$

fancy running average