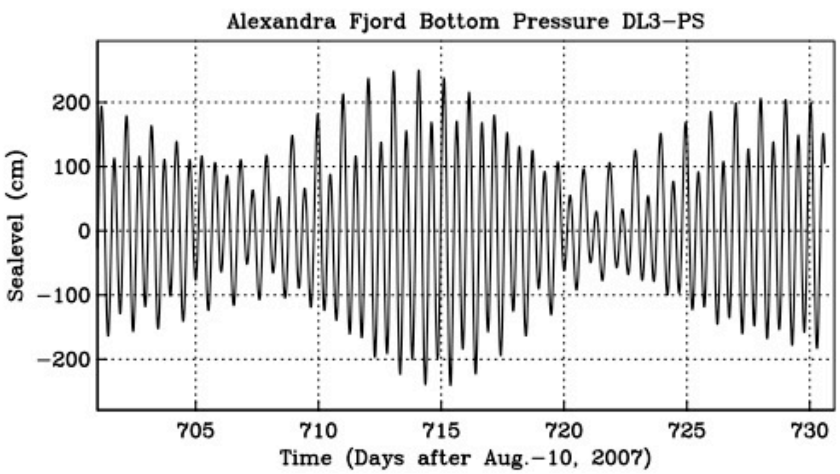
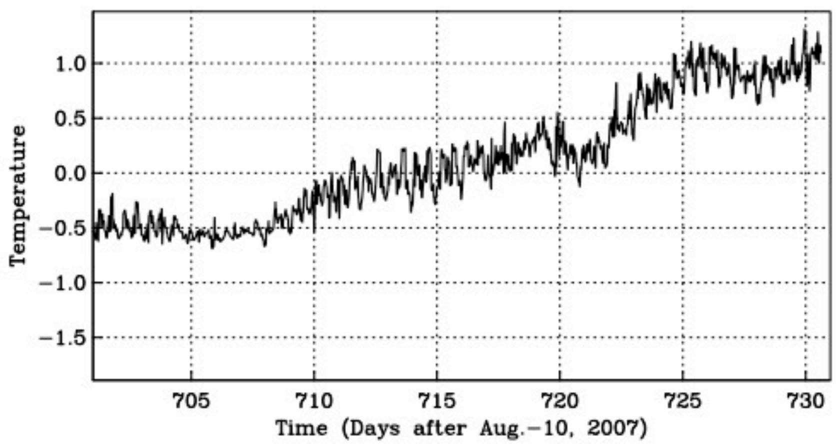
Scalar Time Series Data





Deterministic Data

can be predicted into the future

Example: Tides

Stochastic or Random Data

cannot be predicted into the future
without stating probabilities

Example: Weather

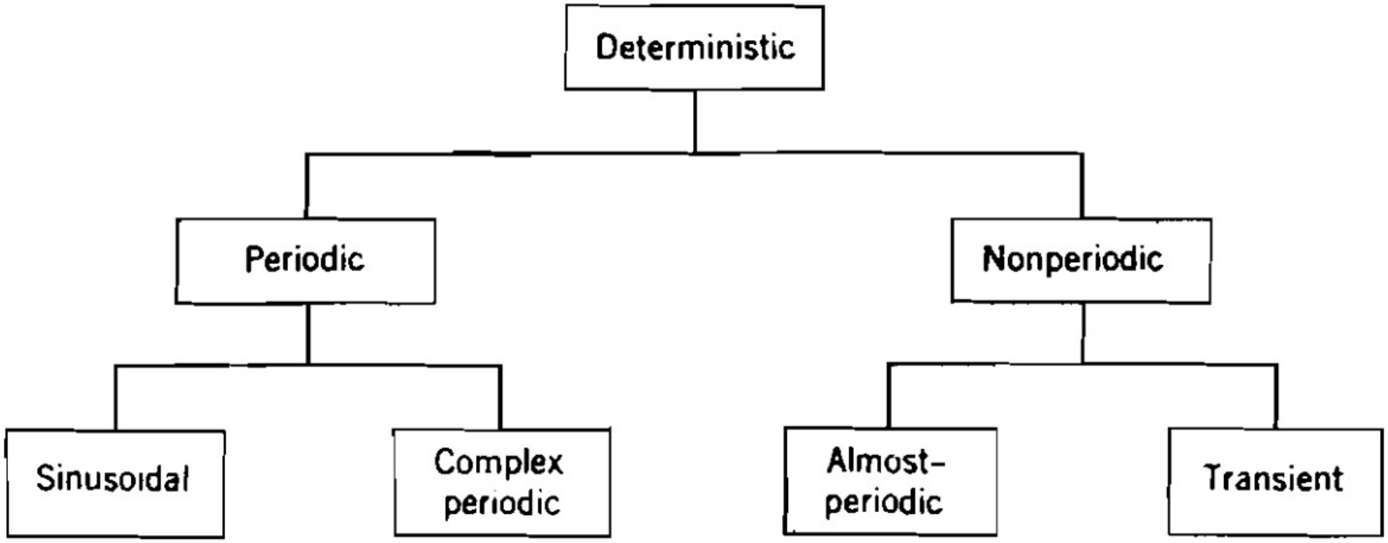


Figure 1.2 Classifications of deterministic data.

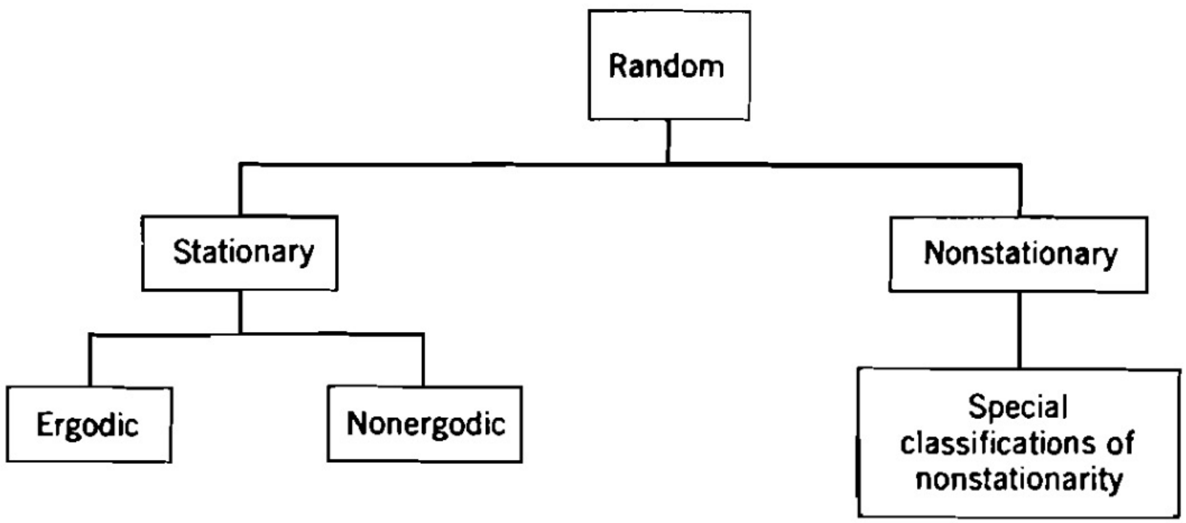


Figure 1.9 Classifications of random data.

Bendat & Piersol (1986)

Time Domain ⇋ Frequency Domain

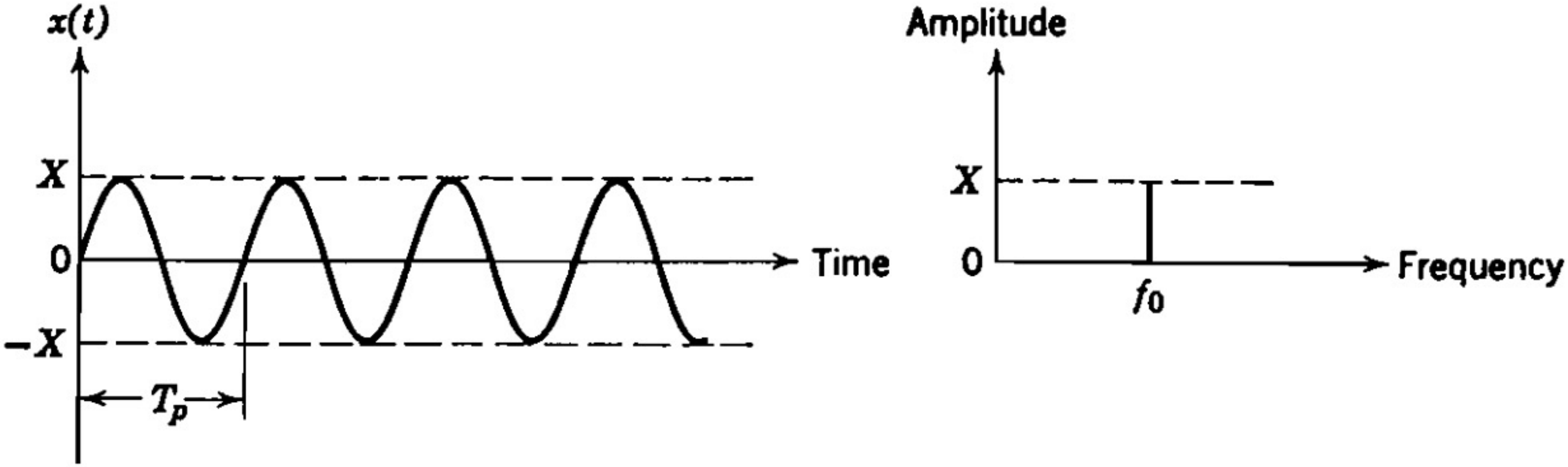


Figure 1.3 Time history and spectrum of sinusoidal data.

$T_p = 1 / f_0$ Period

$f_0 = 1 / T_p$ Frequency

Bendat & Piersol (1986)

$$x(t) = x(t \pm nT_p) \qquad n = 1, 2, 3, \ldots$$

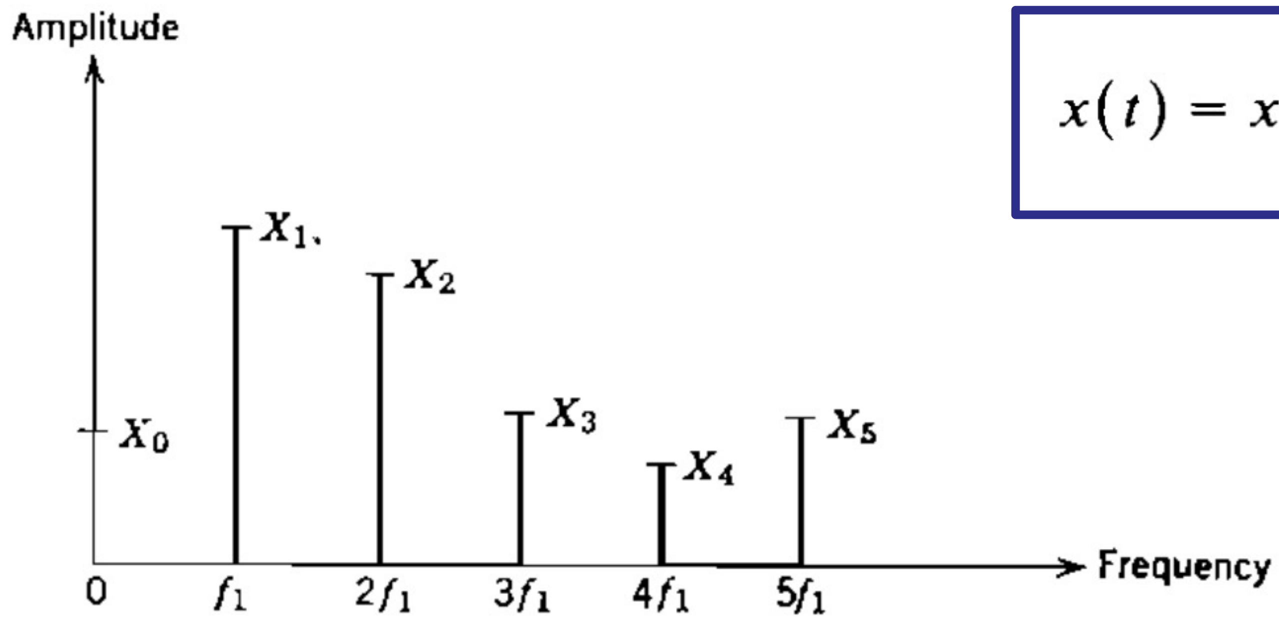


Figure 1.4 Spectrum of complex periodic data.

$$x(t) = \sum_{n=1}^{\infty} X_n \sin(2\pi f_n t + \theta_n)$$

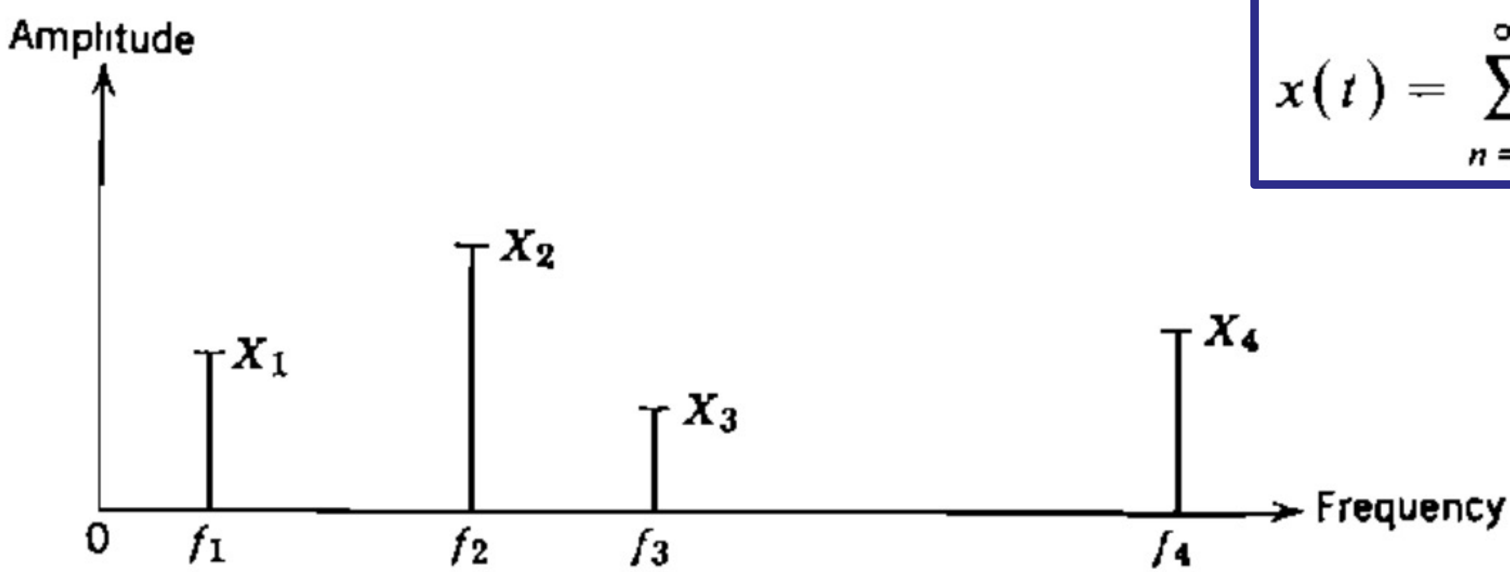


Figure 1.5 Spectrum of almost-periodic data.

Bendat & Piersol (1986)

Time Domain

Frequency Domain

$$x(t) = \begin{cases} A\,e^{-at} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$x(t) = \begin{cases} A\,e^{-at}\cos bt & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$x(t) = \begin{cases} A & c \geq t \geq 0 \\ 0 & c < t < 0 \end{cases}$$

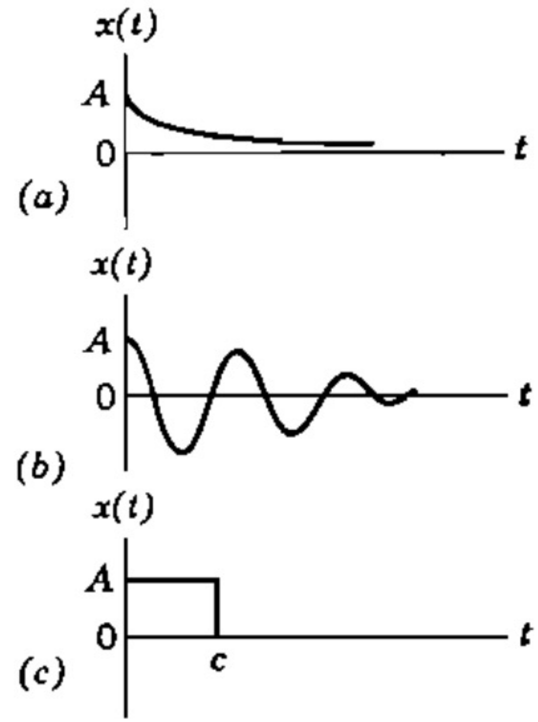
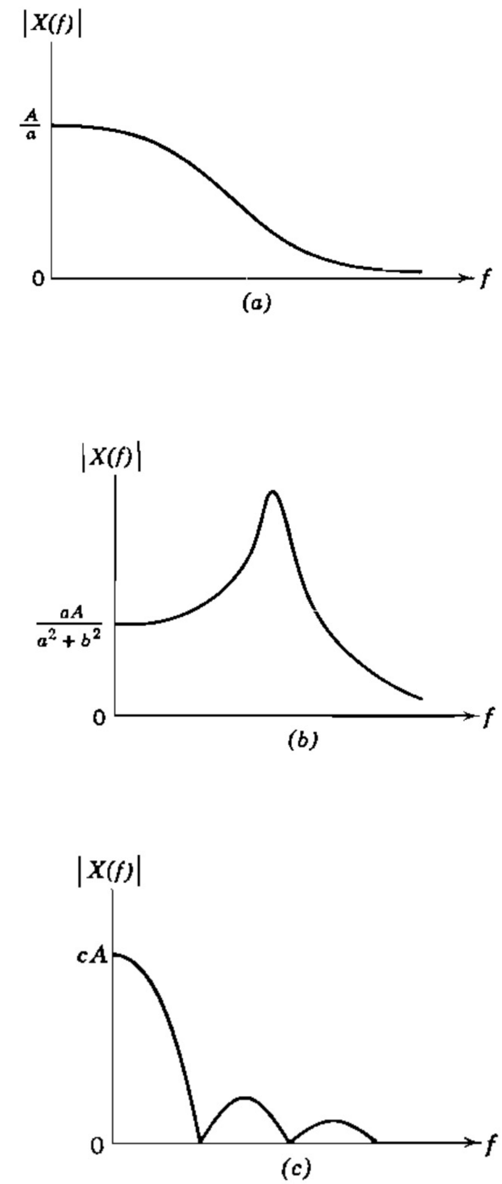


Figure 1.6 Illustrations of transient data.



Bendat & Piersol (1986)

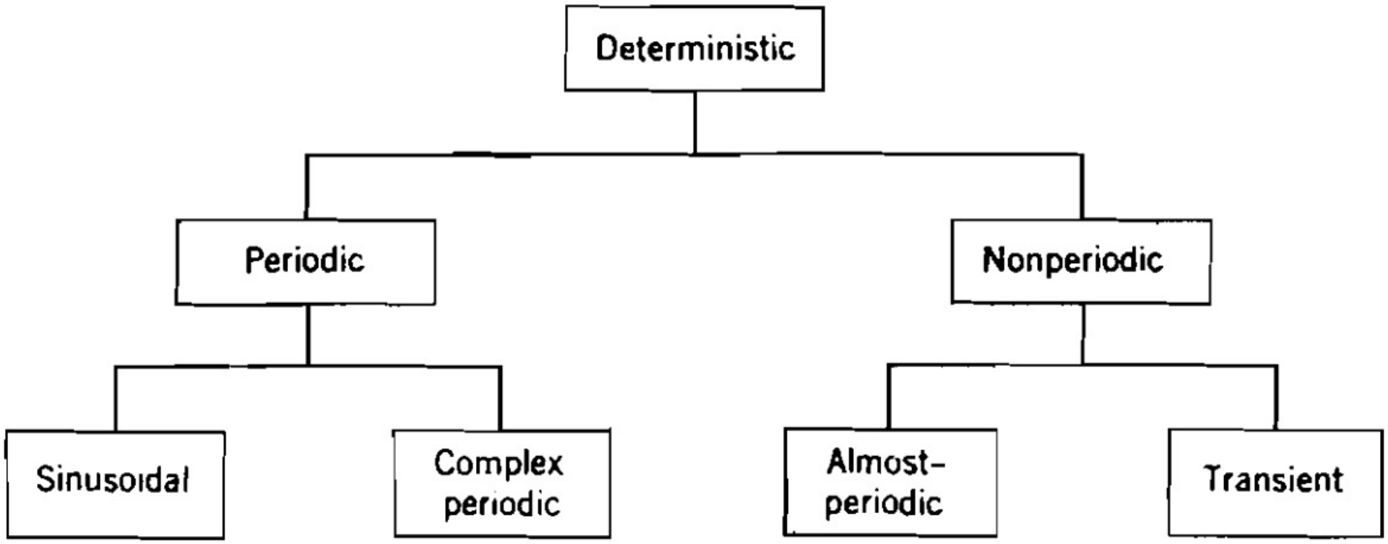


Figure 1.2 Classifications of deterministic data.

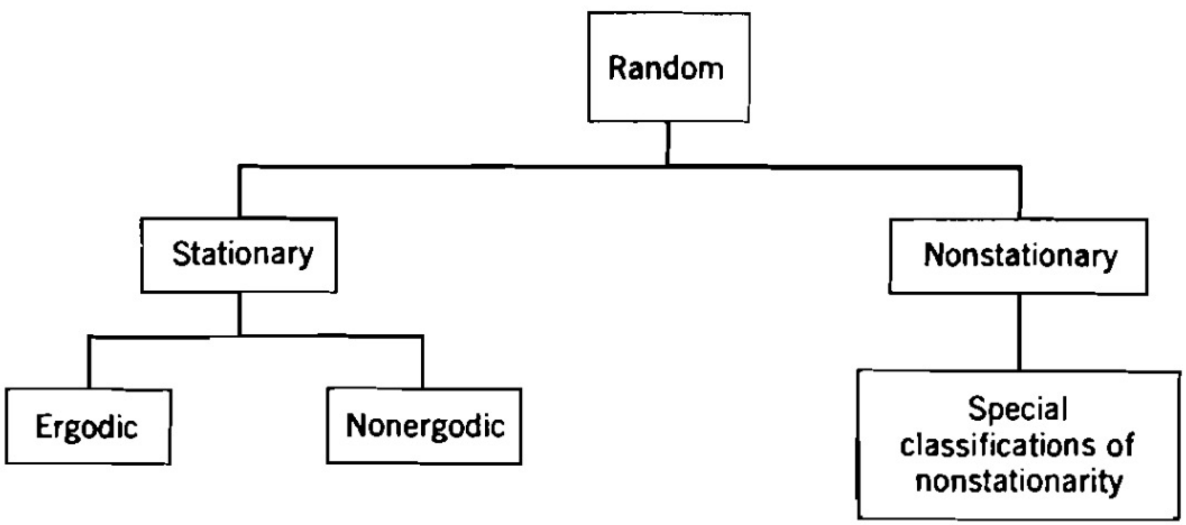


Figure 1.9 Classifications of random data.

Bendat & Piersol (1986)

Ensembles are a set of realizations $x_i(t)$ of a random process $\{x(t)\}$

$x_1(t)$ — Experiment-1
tree-1, drifter-1

$x_2(t)$ — Experiment-2
tree-2, drifter-2

$x_3(t)$ — Experiment-3
tree-3, drifter-3

$x_4(t)$ — Experiment-4
tree-4, drifter-4

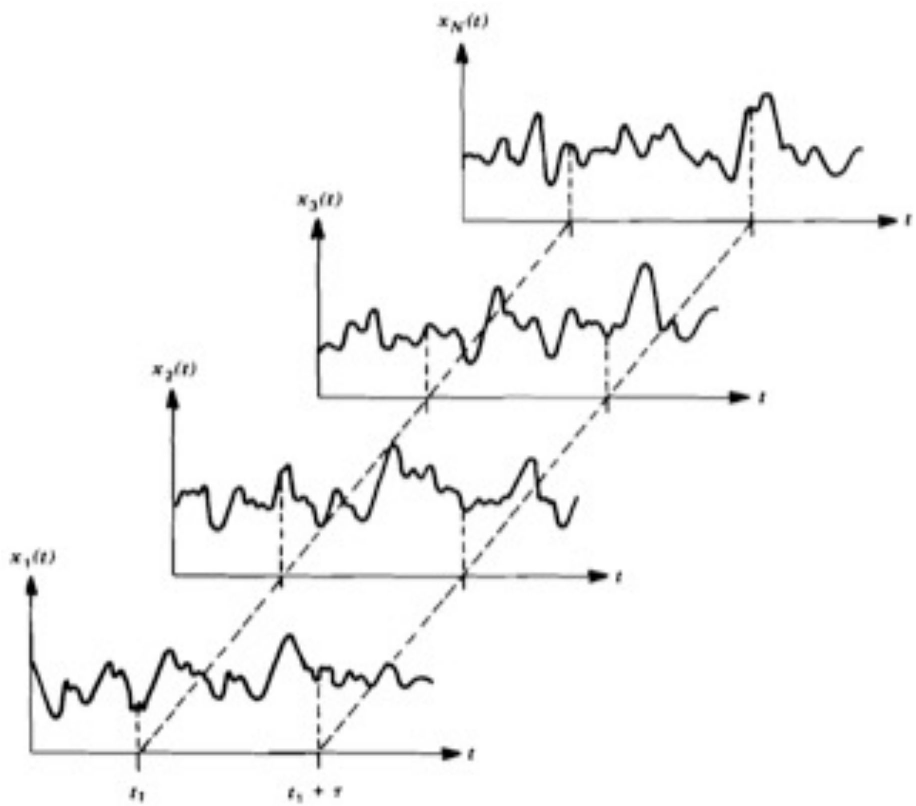


Figure 1.10 Ensemble of time-history records defining a random process.

t ***t+τ*** Time ***t*** and lag ***τ***

B&P Fig. 1.10

Stationary Random Process

Define: Mean over all Ensembles $\mu_x(t)$

$$\mu_x(t_1) = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} x_k(t_1)$$

Define: Auto-Correlation $R_{xx}(t,t+\tau)$

$$R_{xx}(t_1, t_1 + \tau) = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} x_k(t_1) x_k(t_1 + \tau)$$

Bendat & Piersol (1986)

Stationary Random Process

Define: Mean over all Ensembles $\mu_x(t)$
Define: Auto-Correlation $R_{xx}(t,t+\tau)$

If

$$\mu_x(t) = \text{constant}$$

and

$$R_{xx}(t,t+\tau) = R_{xx}(\tau)$$

then

random process $\{x\}$ is stationary.

Ergodic Random Process

Stationary process for which the time average from a single realization can replace an ensemble average.

Go to Lecture Notes or SmartBoard to formalize

MEASUREMENT

=

SIGNAL

+

NOISE

How to separate?

Qualities of time series:

(time domain)

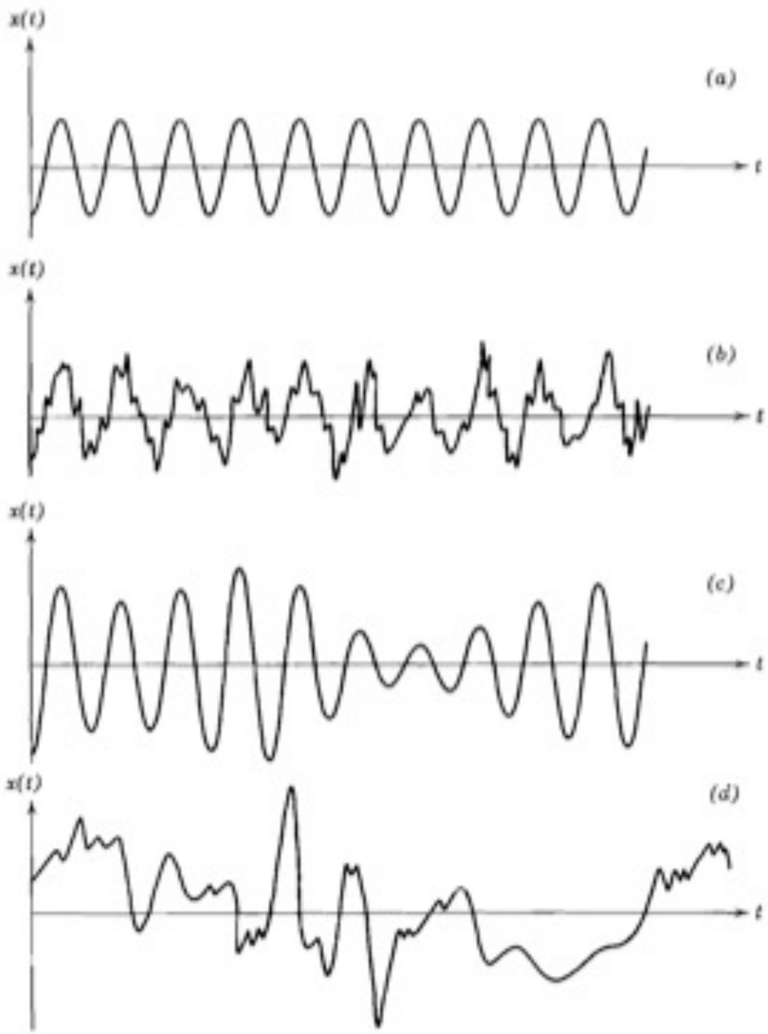


Figure 1.11 Four special time histories. (*a*) Sine wave. (*b*) Sine wave plus random noise. (*c*) Narrow-band random noise. (*d*) Wide-band random noise.

Sine wave

Sine wave plus random noise

Narrow-band random noise

Wide-band random noise

B&P Fig. 1.11

Probability Density Functions:

(histograms)

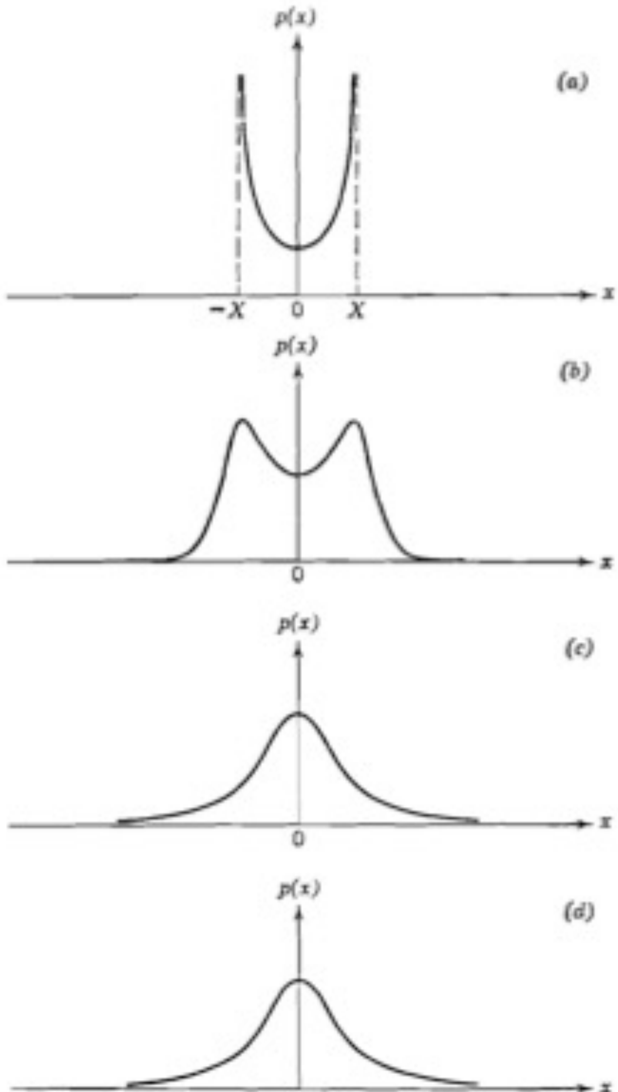


Figure 1.12 Probability density function plots. (*a*) Sine wave. (*b*) Sine wave plus random noise. (*c*) Narrow-band random noise. (*d*) Wide-band random noise.

Sine wave

Sine wave plus random noise

Narrow-band random noise

Wide-band random noise

B&P Fig. 1.12

Auto-correlation functions

(time domain)

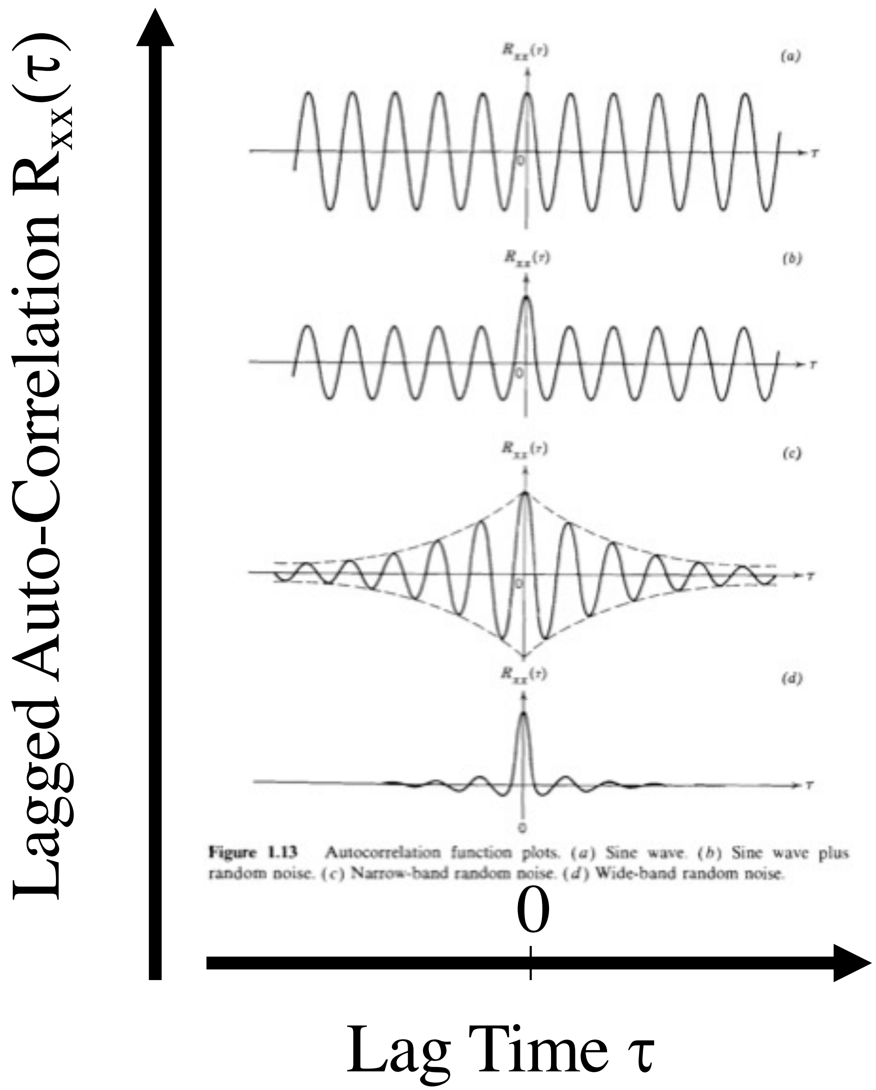


Figure 1.13 Autocorrelation function plots. (*a*) Sine wave. (*b*) Sine wave plus random noise. (*c*) Narrow-band random noise. (*d*) Wide-band random noise.

Sine wave

Sine wave plus random noise

Narrow-band random noise

Wide-band random noise

B&P Fig. 1.13

Auto-spectral Density

(frequency domain)

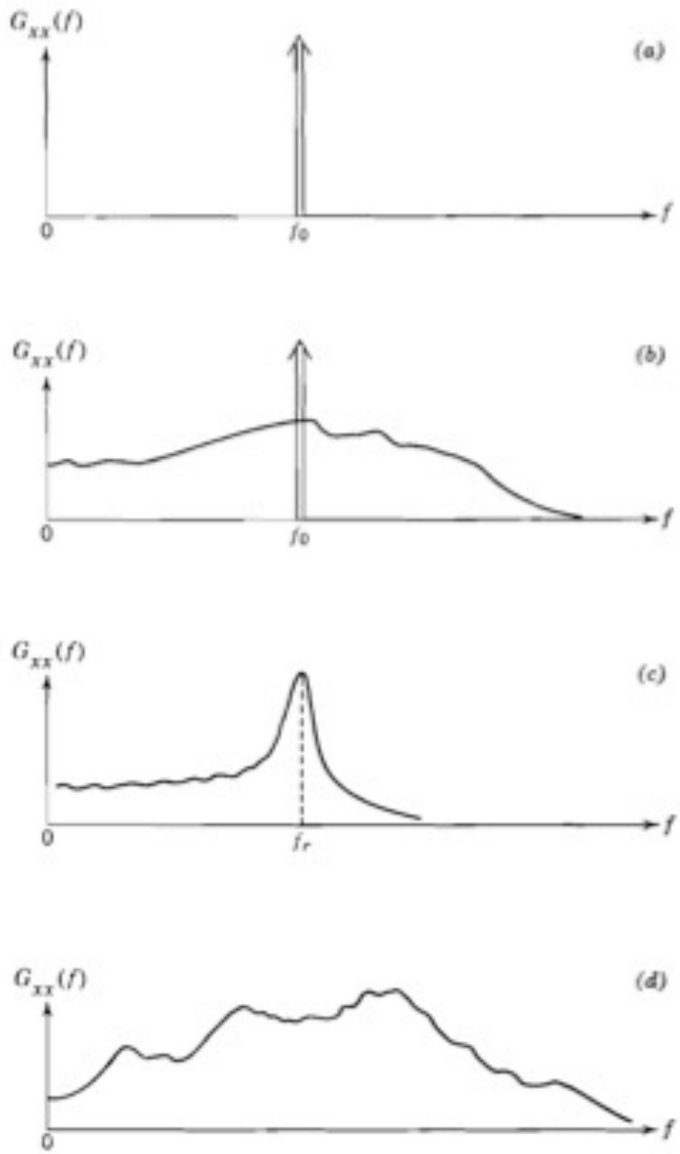


Figure 1.14 Autospectral density function plots. (*a*) Sine wave. (*b*) Sine wave plus random noise. (*c*) Narrow-band random noise. (*d*) Wide-band random noise.

Sine wave

Sine wave plus random noise

Narrow-band random noise

Wide-band random noise

Bendat & Piersol (1986)