Maxwell's Equations

(1) $\vec{\nabla} \cdot \vec{E} = \cancel{\sigma / \varepsilon_0} \; 0$ — Gauss Law for electric fields: electric field induced by charges

(2) $\vec{\nabla} \cdot \vec{B} = 0$ — magnetic monopoles do not exist

(3) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ — Faraday's Law of Induction: electric field inducted by time-varying magnetic field

(4) $\vec{\nabla} \times \vec{B} = \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} + \cancel{\vec{J}} \; 0$ — Ampere's Law modified by Maxwell

$\vec{E}$	electric field intensity	volts / m		
$\vec{B}$	magnetic flux density	weber / m^2 = tesla		
σ	volume charge density	coulomb / m^3	$\sigma = 0$	vacuum, air
$\vec{J}$	electric current density	ampere / m^2	$\sigma \neq 0$	water
ε_0	electrical permittivity	$8.85 \cdot 10^{-12}$ farad / m	$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$	speed of light
μ_0	magnetic permeability	$4\pi \cdot 10^{-7}$ henry / m		

$$\vec{\nabla} = \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{pmatrix}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \quad \text{scalar}$$

$$\vec{\nabla} \times \vec{E} = \begin{pmatrix} \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \\ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \end{pmatrix} \quad \text{vector}$$

Use

$$\vec{\nabla} \times (\vec{\nabla} \times E) = \underbrace{\vec{\nabla}(\vec{\nabla} \cdot \vec{E})}_{\cancel{0 \text{ because of } (1)}} - \vec{\nabla}^2 \vec{E}$$

math, always true

to take curl of (3)

$$-\vec{\nabla}^2 \vec{E} = -\vec{\nabla} \times \frac{\partial \vec{B}}{\partial x}$$

$$= -\frac{\partial}{\partial t}\left(\vec{\nabla} \times \vec{B}\right)$$

use (4)

$$= -\frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E}$$

This then becomes

the wave equation

$$\boxed{\frac{\partial^2 \vec{E}}{\partial t^2} - c^2 \vec{\nabla}^2 \vec{E} = 0}$$

which has plane wave solutions $\vec{E} = \vec{E}_0 \exp\left[i(kz - \omega t)\right]$

subject to the dispersion equation

$k = 2\pi / \lambda$

$\omega = 2\pi / T$

$= 2\pi f$

$i = \sqrt{-1}$

$$k^2 = \underbrace{\omega^2 \mu \varepsilon}_{\text{real}} - i \underbrace{\omega \mu \sigma}_{\text{imaginary}}$$

for vacuum or air $\sigma = 0$ & $k^2 = \omega^2 \mu \varepsilon$

or $c = \dfrac{\omega}{k} = \dfrac{1}{\sqrt{\mu \varepsilon}}$

Same phase speed for all waves

For the ocean, $\sigma \neq 0$, and thus the full dispersion relation must be used

$$\cancel{k^2 = \omega^2 \mu \varepsilon - i \omega \mu \sigma} \quad *$$
$$\cancel{= \omega^2 \mu \varepsilon_0 [\varepsilon / \varepsilon_0 \mp i \sigma / \omega \varepsilon_0]}$$
$$\cancel{= (\omega / c)^2 \, [\varepsilon' \mp i \varepsilon'']}$$

$$k^2 = \omega^2 \mu \varepsilon + i \omega \mu \sigma$$
$$k^2 = \omega \mu \varepsilon_0 \left[\frac{\varepsilon}{\varepsilon_0} + i \frac{\sigma}{\varepsilon_0 \omega} \right]$$
$$k^2 = \left(\frac{\omega}{c}\right)^2 [\varepsilon' + \varepsilon'']$$

or

$$k = \omega / c \, \sqrt{\varepsilon_r}$$
$$= \omega / c \, (n + i \chi)$$

Note that $n = n(\omega)$
$\chi = \chi(\omega)$

index of refraction n depends on the wave frequency (or wavelength)
$\longrightarrow$ dispersive waves

Inserting the so-found wavenumber $k = k(\omega)$ into the wave solution gives a damped wave

$$\vec{E} = \vec{E}_0 \, e^{-\omega \chi z / c} \cdot e^{i(kz - \omega t)}$$

damping   oscillating component

The damping depends on frequency ω and complex part of the index of reflection.

Also, recall that $c = \dfrac{\omega}{k} = \dfrac{\lambda}{T} = \lambda f$

blue	440 nm	$9 \cdot 10^{-10}$	40 m
infrared	10 μm	$5 \cdot 10^{-2}$	16 μm

(*) Eq. 3.3 in the text has the wrong sign

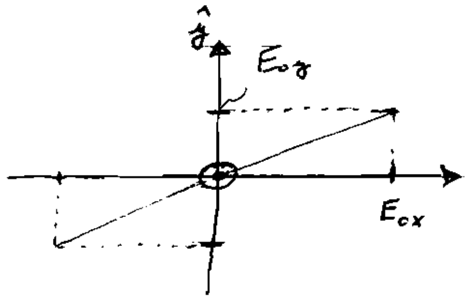
λ χ e-folding da

Polarization

$$\vec{E} = \underbrace{E_{ox}\cos(kz-\omega t)}_{\text{wave in } x,}\,\hat{x} + \underbrace{E_{oy}\cos(kz-\omega t+\phi)}_{\text{wave in } y}\,\hat{y}$$

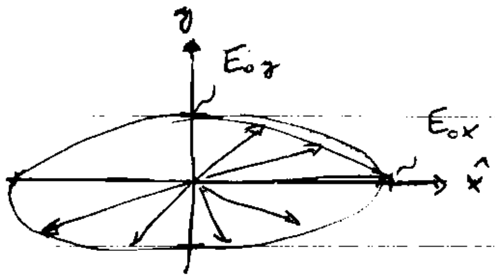
propagating in the z direction

linear polarized



$\phi = 0$, the x and y components of the wave traveling in z are in phase

elliptically polarized



$\phi = \pi/2$, the x and y components are out of phase, $\vec{E}$ rotates around the propagation direction

for $E_{ox} = E_{oy}$ and ϕ arbitrary $\longrightarrow$ circular polarized

H-pole and V-pole components are E_x and E_y in an earth-referenced co-ordinate system

Most general wave solutions $\vec{E}(\vec{r},t) = \vec{E}_o \cos(\vec{k}\cdot\vec{r} - \omega t)$

where $\vec{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$ and $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

Radiant Flux Φ: energy transported from/to a surface — Watts

$$\Phi_{sun} = 3.9 \cdot 10^{26} \text{ Watts}$$

Radiant Intensity $J = \frac{\Delta \Phi}{\Delta \Omega}$: radiant flux per unit solid angle — Watts/sr

$$J_{sun} = \Phi_{sun} / 4\pi = 3.1 \cdot 10^{25} \text{ W/sr}$$

Flux Density $\frac{\Delta \Phi}{\Delta A}$: radiant flux per unit area — Watts/m²

SUN | · earth

$1.5 \cdot 10^{8}$ km $= 1.5 \cdot 10^{11}$ m

↳ 1 m² area corresponds to $4.4 \cdot 10^{-23}$ sr

↳ 1400 W/m²

$$\Delta A = r^2 \sin\theta \, \Delta\theta \, \Delta\phi$$

$$\frac{1}{1.5^2 \cdot 10^{22}} =$$

Radiance $L = \frac{d^2 \Phi}{d\Omega \, dA \cos\theta}$ — radiant flux to or from a surface in a specified direction — W/m²/sr

Spectral forms of radiant flux $\frac{\Delta \Phi}{\Delta \lambda}$ — W/μm

$\frac{\Delta L}{\Delta \lambda}$ — W/m³/sr

Absorption and Emission

Surfaces interact with radiation in 4 ways

(a) emit
(b) absorb
} real objects less efficient than black body; directional dependence
(c) reflect
(d) transmit

$$e = \text{emissivity} = \frac{\cancel{\text{gray body}}\ \text{radiance}}{\text{black body}} = \frac{L_\lambda(\lambda, T, \theta, \phi)}{f_p(\lambda, T)} \in [0, 1]$$

T - temperature

$$f_p = 2\hbar c \Big/ \lambda^5 e^{(\hbar c / k_B \lambda T - 1)}$$

$W/m^3/sr$

$e = e(\lambda)$

1st Example: infrared, $\theta \leq 45°$

$e_{ice} \approx e_{water} \approx 0.98$ $\quad e_{water} \approx 0.98$ $\quad e_{ice} \approx 0.98$

microwave, $\theta \approx 50°$

$e_{water} \approx 0.4$ $\quad e_{ice} \approx 0.8$

→ @ microwave frequencies sea ice thus can have larger brightness temperatures than water

2nd e can be regarded as a property of the surface somewhat independent of T

→ we can distinguish "material" at pixels based on their e