Lecture #3
March 16, 2010

Top of Atmosphere (TOA) Reflectance ρ_t is

~~Esaa~~ Esaias et al (1998):

$$\rho_t = \underset{\substack{\text{Rayleigh scatter} \\ \text{(molecules)}}}{\rho_r} + \underset{\text{aerosol}}{\rho_a} + t\underset{\substack{\text{waterleaving} \\ \text{reflectance}}}{\rho_w} + \underset{\substack{\text{interaction} \\ \text{of aerosol} \\ \text{+ molecules}}}{\rho_{ra}} + t\underset{\text{whitecaps}}{\rho_{wc}} + \underset{\text{sun glint}}{T\rho_g}$$

where $t = t(\lambda)$ is diffusive transmittance

Wang + Shi (2009): $\rho_t = \rho_r + \rho_a + t\,\rho_w$

ρ_r: look up tables using
- solar-sensor geometry
- atmospheric pressure
- wind speed

ρ_a: two NIR bands, bands 15 + 16 @ 748 nm, 869 nm

TAO Rayleigh-corrected NIR reflectance $\Delta\rho^{(RC)} = \underset{\substack{\text{sensor} \\ \text{measured}}}{\rho_t} - \underset{\substack{\text{look up} \\ \text{table}}}{\rho_r}$

6.1 Introduction (Peixoto + Oort, 1992, Physics of Climate)

Solar Radiation: • Incoming EM waves with speed c

$$c = \lambda \cdot \nu \approx 3 \cdot 10^8 \text{ m/s}$$

λ wavelength
ν frequency
= 1/waveperiod

- partly absorbed
- partly scattered
- partly reflected

} by atmospheric gases, aerosols, clouds

<u>steady state</u>: absorbed energy equals energy moving outer space

incoming solar = outgoing terrestrial
+ outgoing solar reflected

$\boxed{\text{Figure 6.3}}$ Schematic of Global Radiation Budget

$\boxed{\text{Figure 6.1}}$ Spectral distribution of solar radiation @ top of atmosphere
@ sea surface

$J = J(\lambda)$

Energy = h · frequency

6.2 Physical Radiation Laws

(a) **Planck:** Energy emitted by black body is uniquely determined by its temperature T at each frequency (or wavelength).

$$\text{Intensity} = \frac{\text{Energy}}{\text{time} \cdot \text{area} \cdot \text{angle}} = B_\nu(T) = \frac{2 h \nu^3}{c^2 \left(e^{h\nu/kT} - 1\right)}$$

where $h = 6.6 \cdot 10^{-34}$ J s Planck's constant

$k = 1.4 \cdot 10^{-23}$ J / K Boltzmann's constant

$c = \lambda \cdot \nu$ speed of light

(b) Stefan-Boltzmann: The total intensity emitted by a black body is proportional to T^4

$$B(T) = \int_0^\infty B_\nu(T)\, d\nu \quad \propto \quad T^4$$

substitute $u = \frac{h\nu}{kT}$ $\quad du = \frac{h}{kT}\, d\nu$

$$= \underbrace{\frac{2}{h^3 c^2} k^4}_{\text{constant}} \cdot \underbrace{\int_0^\infty \frac{u^3}{e^u - 1}\, du}_{\text{fancy integral to be solved by Residues (Fermi Integral)}} \cdot T^4 \quad \propto \quad T^4$$

$$\int_0^\infty B_\lambda(T)\,d\lambda = \int_0^\infty \frac{2\hbar c}{\lambda^5\left(e^{c\hbar/k\lambda T}-1\right)}\,d\lambda \propto T^4$$

I do not see this

$$\int_0^\infty B_\nu(T)\,d\nu = \int_0^\infty \frac{2\hbar\nu^3}{c^2\left(e^{\hbar\nu/kT}-1\right)}\,d\nu \propto T^4$$

$$u = \frac{\hbar\nu}{kT} \qquad \frac{du}{d\nu} = \frac{\hbar}{kT} \qquad \Rightarrow du = \frac{\hbar}{kT}\,d\nu$$

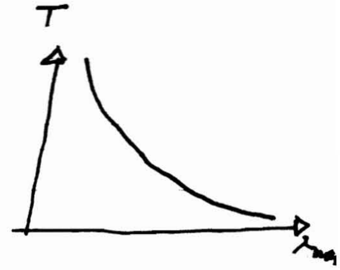
$$\Rightarrow \nu = \frac{u\cdot k\cdot T}{\hbar}$$

$$\Rightarrow \quad \nu^3 = \frac{u^3k^3T^3}{\hbar^3} \qquad d\nu = \frac{kT}{\hbar}\,du$$

$$= \int_0^\infty 2\hbar\,\frac{k^3T^3u^3}{\hbar^3}\cdot\frac{1}{c^2\left(e^{\cancel{\hbar}\frac{u\cancel{kT}}{\cancel{\hbar}}\cdot\frac{1}{\cancel{kT}}}-1\right)}\cdot\frac{kT}{\hbar}\,du$$

$$= \frac{2}{\hbar^3c^2}\,k^4T^4\underbrace{\int_0^\infty\left(\frac{u^3}{e^u-1}\right)du}_{\text{fancy integral to be solved by Residues (Fermi integral)}} \propto T^4$$

(c) Wien's Displacement Law: The wavelength of max. emission of a black body is inversely proportional to its Temperature T.



find a maximum: $\dfrac{\partial B(T,\lambda)}{\partial \lambda} = 0$

$\Rightarrow \quad \lambda_{max} = \text{const}/T \qquad$ const. $\approx 3000\,\mu m\ K$

$T \approx 293\ K$ (earth's surface) $\Rightarrow \lambda_{max} \approx 10\,\mu m$ infrared

$T \approx 6100 K$ (sun's surface) $\Rightarrow \lambda_{max} \approx 0.47\,\mu m$ green

Note that Stefan-Boltzmann and Wien's Laws are special cases or derivatives of the more general Planck's Law.

(d) Kirchhoff's Law

monocromatic (at each wavelength) intensity of radiation J_λ

$$\frac{J_{\lambda a}(\lambda)}{J(\lambda)} + \frac{J_{\lambda r}(\lambda)}{J(\lambda)} + \frac{J_{\lambda \tau}(\lambda)}{J(\lambda)} = 1$$

absorptivity	reflectivity	transmissivity
$\in [0,1]$	$\in [0,1]$ "albedo"	$\in [0,1]$

$$a(\lambda) + r(\lambda) + \tau(\lambda) = 1$$