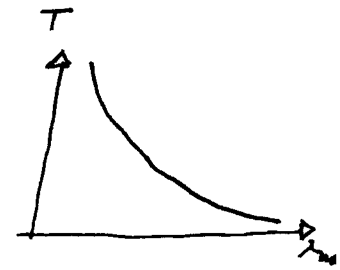
(c) Wien's Displacement Law: The wavelength of max. emission of a black body is inversely proportional to its Temperature T.

find a maximum: $\dfrac{\partial B(T,\lambda)}{\partial \lambda} = 0$



$\Rightarrow \quad \lambda_{max} = const/T$ $\qquad$ const. $\approx 3000\,\mu m$ K

$T \approx 293$ K (earth's surface) $\Rightarrow$ $\lambda_{max} \approx 10\,\mu m$ infrared

$T \approx 6100$K (sun's surface) $\Rightarrow$ $\lambda_{max} \approx 0.47\,\mu m$ green

Note that Stefan-Boltzmann and Wien's Laws are special cases or derivatives of the more general Planck's Law.

start

(d) Kirchhoff's Law $\qquad J(\lambda) = J_a(\lambda) + J_r(\lambda) + J_\tau(\lambda)$

total $\quad$ absorbed $\quad$ reflected $\quad$ transmitted

monocromatic (at each wavelength) intensity of radiation J_λ

$$\frac{J_{\lambda a}(\lambda)}{J(\lambda)} + \frac{J_{\lambda r}(\lambda)}{J(\lambda)} + \frac{J_{\lambda \tau}(\lambda)}{J(\lambda)} = 1$$

absorptivity $\in [0,1]$ $\quad$ reflectivity $\in [0,1]$ "albedo" $\quad$ transmissivity $\in [0,1]$

$$a(\lambda) + r(\lambda) + \tau(\lambda) = 1$$

For a black body $a(\lambda) = \frac{\text{absorbed}}{\text{total}}$ intensity $= \underline{1}$ → $r(\lambda) = \tau(\lambda) = 0$

→ no reflected or transmitted energy

Kirchhoff's Law: A surface in thermodynamic equilibrium (steady state, no time change) with its surrounding must absorb and emit energy at the same rate:

$$\cancel{a_\lambda = \varepsilon_\lambda} \qquad a(\lambda) = \varepsilon(\lambda)$$

where $\varepsilon(\lambda) = \frac{J(\lambda)}{B(\lambda)} = \frac{\text{emitted intensity}}{\text{Planck's intensity}}$ = emissivity

(1) use example $\varepsilon = \varepsilon(\lambda)$:
infrared $\varepsilon_{water} \approx \varepsilon_{ice}$
microwave $\varepsilon_{water} \approx 1/2\ \varepsilon_{ice}$
→ @ microwave sea ice thus can have larger brightness temp. than water

because if it were not, then the temperature would change which is not allowed in a closed (steady state) system in thermodynamic equilibrium.

(2) ε is a property of the surface somewhat independent of T → distinguish "material" (clouds, ice, water) based on their $\varepsilon(\lambda)$

(e) Beer-Bouguer-Lambert law: How does radiation intensity $J(\lambda)$ change due to absorption?

e.g., sun light through atmosphere along layer dz

$$dJ_\lambda = -k_{\lambda a} \cdot J_\lambda \cdot \rho \cdot dz$$

$k_a = k_a(\lambda)$
$J = J(\lambda)$

change in intensity — absorption coefficient — density of medium — layer thickness

Write as differential equation and solve as such

$$\frac{dJ}{dz} = - k_a \cdot J \cdot \rho$$

if medium (atmosphere, say) is homogeneous $\rightarrow \rho = const$
if absorption coefficient is independent of z $\rightarrow k_a = const$ but $k_a = f(\lambda)$

Then $$\int_{J_0}^{J} \frac{1}{J} dJ = - \rho k_a \int_{z=0}^{z} dz$$

or

$$J(z) = J(z=0)\, e^{-\rho k_a z}$$

Similar argument / law for "scattering" with a scattering coefficient k_s

$$J(z) = J(z=0)\, e^{-\rho k_s z}$$

Or if both absorption and scattering occurs (as it does in atmosphere)

$$J(z) = J_0\, e^{-\rho k z}$$ $k = k_a + k_s$
extinction = absorption + scattering

Transmissivity then becomes

$$\tau(\lambda) = \frac{J(\lambda, z)}{J(\lambda, z=0)} = e^{-\int_0^\infty k \rho\, dz}$$

$k \neq const$
$\rho \neq const$

$k = k(\lambda, z)$
$\rho = \rho(z)$

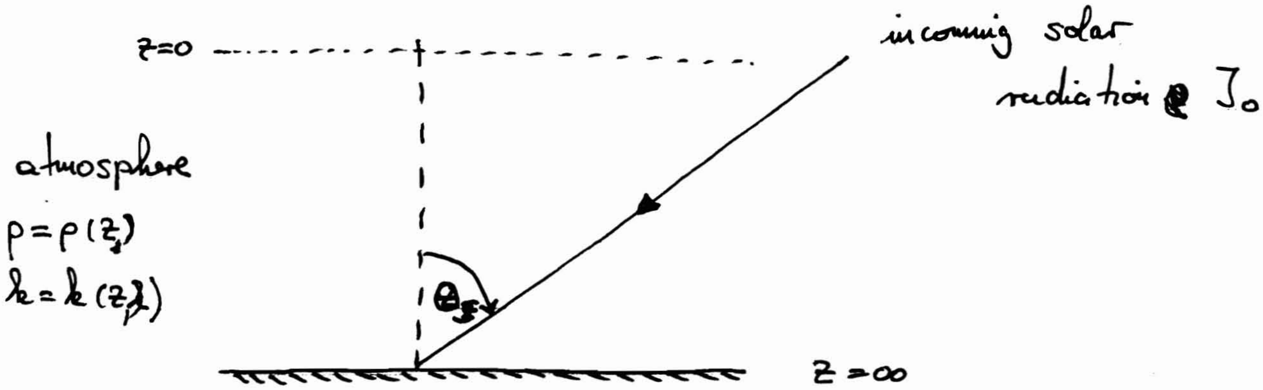
The quantity in the exponent

$$-\int_0^\infty k \rho \, dz = u \qquad \text{non-dimensional}$$

is called "optical depth" or "optical thickness."

transparent ~~atmosphere~~	$u = 0$	↕ transmissivity	$\tau = e^{-0} = 1.00$	incoming J_0 reduced 0%
atmosphere	$u = 1$	↕ transmissivity	$\tau = e^{-1} = 0.37$	incoming J_0 reduced 63%
heavy clouds	$u = 2$	↕ — " —	$\tau = e^{-2} = 0.14$	incoming J_0 reduced 86%

Further complication due to solar zenith angle Θ_s



which adds geometric factor $\sec \Theta_s = \dfrac{1}{\cos \Theta}$ (all the above is for $\Theta_s = 0$)

6.3 Solar Radiation

Incoming energy:	$\lambda < 0.4\ \mu m$	9%	$\lambda \in [0, 400]$ nm
	$0.8 > \lambda > 0.4\ \mu m$	49%	$\lambda \in [400, 800]$ nm
	$0.8 < \lambda$	42%	$\lambda \in [800, \infty]$ nm

Measurement indicate a solar constant of radiation reaching (TOA) the earth

$$S = 1360\ W/m^2$$

$\frac{\text{earth cross section}}{\text{earth surface area}} = \frac{\pi R_E^2}{4\pi R_E^2} = 1/4$

Spectrum @ top of the atmosphere (TOA) resembles Planck's Law for $T \approx 6100$ K

$\boxed{\text{FIG 6.1}}$ solar irradiation at TOP and at sealevel for $\Theta_S = 0$
sun vertical overhead

Solar radiation at TOA depends on

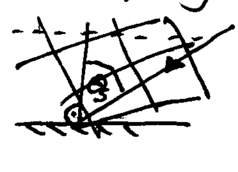
(a) geometry of globe
(b) rotation of globe
(c) elliptical orbit around sun

} tilt of ecliptic plane $23°27'$
eccentricity of orbit 0.0167
longitude of perihelion

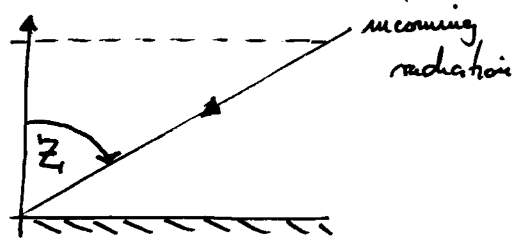
(d) mean distance sun-earth $1.496 \cdot 10^{11}$ m $= 1$ AU
astronomical unit

absorbed radiation is directly added to the heat budget
scattered radiation is partly returned to space, partly continues through atmosphere with further absorption and scattering

~~irradiance~~ $\cancel{F_{SW} = F^0_{SW} \cos\Theta_S}$



Irradiance F_{SW} depends on solar zenith angle $Z_1 \equiv \theta_S$ (both notations used in lit.)



$$F_{SW} = F_{SW}^{\circ} \cos Z_1 \qquad \text{where } F_{SW}^{\circ} = S\left(\frac{d_m}{d}\right)^2$$

$S = 1360 \ \mathrm{W/m^2}$

$d_m = 1 \ \mathrm{AU}$

$d = d(\text{lat}, \text{long}, \text{time})$ actual distance

Total daily insolation at TOA

$$Q_o = \int_{\text{sunrise}}^{\text{sunset}} S(d_m/d)^2 \cos Z_1 \, dt$$

$\boxed{\text{FIG. 6.4}}$ $\quad Q_o = Q_o(\text{latitude}, \text{month})$

Radiation is absorbed and scattered by

Aerosols	solid particles in air 10^{-4} to 10^{+1} μm	dust, soot, sea salt, pollution, volcanos, fires, NO_x, SO_2, HSO_4, etc.
atmospheric gases	water vapor, CO_2, ozone	
clouds		