MAST-693 and CIEG-693 Homework #2 (due Mar.-18, 2020)

1. [5 pts] In D = 1 m and D = 10 m water depth, what frequency $f=\omega/(2\pi)$ (in Hz) corresponds to kD=0.1, kD=1, and kD=10 from the full dispersion relationship? Make a 3 by 2 element table.

2. [10 pts] Plot the non-dimensional dispersion relationship $\omega^2 D/g = kD \tanh(kD)$ versus kD. Then plot the shallow water approximation to this. At what kD is the shallow water approximation in 20% error?

3. [10 pts] The shallow water approximation to the non-dimensional dispersion relationship is

$$\omega^2 D/g = (kD)^2.$$

Derive the next higher order in kD dispersion relationship from the full dispersion relationship. What is the corresponding phase speed c?

4. [10 pts] In the shallow water approximation:

(a) Write out the expression for u as a function of a, h, and c.
(b) What non-dimensional parameter comes out of the ratio of u/c?
(c) What limitations on size does this parameter have?

5. [5 pts] In 5-m water depth and a wave of period T= 18 s and wave height H=2a=1 m.

(a) Do you think that shallow water approximation is valid given results from above?
(b) What is the magnitude of u?

6. [5 pts] In water depth D, suppose pressure

$$p_w(x,z,t) = \rho\, a\, \omega^2\, k\, \{\cosh[k(z+D)] / \sinh(kD)\} \cos(kx-\omega t)$$

and horizontal velocity

$$u(x,z,t) = a\, \omega\, \{\cosh[k(z+D)]/\sinh(kD)\} \cos(kx-\omega t)$$

are measured at the same vertical location z. Derive an expression for $p_w/(\rho u)$.

7. [5 pts] In water depth D, suppose a pressure sensor measures pressure $p_w(x,z= z_p,t)$ at vertical location z_p and a current meter measures u at a different vertical location z_u, that is, $u(x,z= z_u,t)$. In real life, this often is the case.

(a) Given $p_w(z_p,t)$, give an expression for wave pressure at z_u.
(b) Then write an expression for the ratio of $(p_w)/(\rho u)$ at z_u using p_w measured at z_p and u measured at z_u.