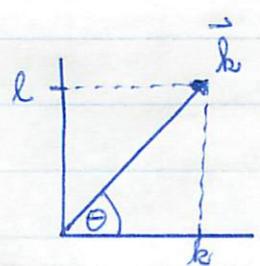


$L_x = \frac{\sigma}{k} \cdot T$ is the distance in the x -direction that the wave propagates in the x -direction in one period $T = 2\pi / \sigma$

$\lambda = |\vec{C}_p| \cdot T$ is the distance that the wave propagates its phase along its path in one wave period. This is the wavelength
 $= 2\pi / |\vec{k}|$ is equal to 2π divided by the magnitude of the wave number vector, i.e., $|\vec{k}| = (k^2 + l^2)^{1/2}$

$$\begin{aligned} |\vec{k}| &= k \cos \theta \\ |\vec{k}| &= l \sin \theta \end{aligned} \quad \text{or} \quad \vec{k} = \begin{pmatrix} k \\ l \end{pmatrix} = |\vec{k}| \begin{pmatrix} 1 / \cos \theta \\ 1 / \sin \theta \end{pmatrix}$$



$$\vec{C}_p = \frac{\sigma}{|\vec{k}|} \cdot \frac{\vec{k}}{|\vec{k}|} = \frac{1}{|\vec{k}|} \begin{pmatrix} \sigma / \cos \theta \\ \sigma / \sin \theta \end{pmatrix}$$

Standing Wave is superposition of two oppositely traveling plane waves

$$A e^{i(\vec{k}\vec{x} - \sigma t)} + A e^{i(-\vec{k}\vec{x} - \sigma t)} = 2A e^{-i\sigma t} \cos(\vec{k}\vec{x})$$

1.2 Dispersion $\sigma = \sigma(\vec{k})$ reflects physics
linear

Linear Wave Equation Solution Dispersion

non-dispersive
all waves
travel at
same speed
 $c = \sigma/|\vec{k}|$

- (a) $\phi_t + c\phi_x = 0$ $e^{i(kx - \sigma t)}$ $\sigma = ck$
- (b) $\phi_{tt} - c^2\phi_{xx} = 0$ $e^{i(kx - \sigma t)}$ $\sigma^2 = c^2 k^2$
- (c) $\phi_t + \vec{c} \cdot \nabla\phi = 0$ $e^{i(\vec{k}\vec{x} - \sigma t)}$ $\sigma = \vec{c} \cdot \vec{k}$
- (d) $\phi_{tt} - c^2 \nabla^2\phi = 0$ $e^{i(\vec{k}\vec{x} - \sigma t)}$ $\sigma^2 = c^2 |\vec{k}|^2$

Dispersive
 $c = c(\vec{k})$

(e) $\nabla^2\phi_t + \beta\phi_x = 0$ $e^{i(\vec{k}\vec{x} - \sigma t)}$ $\sigma^2 = \frac{-\beta k_x}{k^2 + l^2 + m^2}$

approximate statement of dynamical and thermodynamical conservation laws

Discuss dispersive vs. non-dispersive waves

- non-dispersive waves "stick" together and travel together from a generation site
- dispersive waves will break up and "disperse" because some waves (shorter or longer) will move their phase faster or slower than others
- standing waves possible only for (b) and (d) as waves of the same wave number can have opposite signs for frequency, that is

$$\omega = \omega_j(\vec{k}) \quad j = 1, 2, \dots, n \quad (\text{multiple branches})$$

- no standing waves for (a), (c), and (e)

$$\rightarrow \text{dispersion relation } \omega = \omega(\vec{k}) \quad \text{single branch}$$

CAUTION

* Plane waves rarely complete solutions, because

(1) \rightarrow consider initial and boundary conditions

(2) medium may not be steady or homogeneous

(3) nonlinear dynamics

1.3 Linear superposition of plane waves

homogeneous medium + initial value problem

solve via Fourier Integrals

If dispersion relation has n branches

$$\sigma = \sigma_j(\vec{k}) \quad j = 1, 2, \dots, n$$

then

$$\phi(\vec{x}, t) = \sum_{j=1}^n \underbrace{\int \int \int_{-\infty}^{+\infty} A_j(\vec{k}) e^{i[\vec{k}\vec{x} - \sigma_j(\vec{k})t]} d\vec{k}}_{\substack{\text{integral over all} \\ \vec{k} = (k, l, m)}}$$

where $A_j(\vec{k})$ is fixed by initial conditions.

For $n=1$ (one dimension) $\sigma = \sigma(k)$

$$\phi(x, t) = \int_{-\infty}^{+\infty} A(k) e^{i[kx - \sigma t]} dk$$

$A(k)$ is fixed by specifying $\phi(x, t=0)$:

$$\phi(x, 0) = \int_{-\infty}^{+\infty} A(k) e^{ikx} dk \quad \Downarrow \quad A(k) = \int_{-\infty}^{+\infty} \phi(x, 0) e^{-ikx} dx$$

If $v = ck$ then

$$\phi(x, t) = \int_{-\infty}^{+\infty} A(k) e^{i(kx + \frac{ck}{k}t)} dk = \int_{-\infty}^{+\infty} A(k) e^{ik(x-ct)} dk = \phi(x-ct, 0)$$

↳ The initial condition $\phi(x, t=0)$ translates towards $x > 0$ at speed c without changing shape.

For homogeneous media

I Find dispersion relation

II Deduce $A_j(\vec{k})$ from initial condition

III Evaluate a set of Fourier integrals