

## Energetics of Acoustic Waves

kinetic energy  
(per unit volume)  $\frac{1}{2} \rho_0 (u^2 + v^2 + w^2)$  kg/m s<sup>2</sup>

Need "potential" restoring energy density (= energy / volume)  
to balance this.

Transport of energy across any plane ( $x = \text{const}$ )

is the rate of working or product of

1. force acting on plane per unit area  $\rightarrow p$

2. velocity component in that direction  $\rightarrow u$

This gives acoustic intensity

$$J = (p - p_0) \cdot u$$

neglect contributions of  $p_0$

How does this work?

A fluid element is compressed, volume changes

$$V \longrightarrow (V - dV)$$

The work done by this compression is

$$p \cdot (-dV) \quad \frac{\text{kg}}{\text{s}^2 \text{m}} \cdot \text{m}^3$$

This represents an energy in Joules =  $\frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \text{m} = \frac{\text{kg}}{\text{s}^2 \text{m}} \cdot \text{m}^3$

The volume per unit mass is

$$\rho^{-1}$$

and thus the work done by the compression ( $-dV$ )

per unit mass is

$$p \cdot (-dp)^{-1} = \frac{p}{\rho^2} dp$$

And the change in "potential" energy of this compression

$$dE^* = \frac{(p-p_0)dp}{\rho_0^2}$$

due to the perturbation  
per unit mass

The energy density (= energy per unit volume) of this change

$$dE_p = \rho_0 dE^* = (p - p_0) \frac{dp}{\rho_0}$$

$$dE_p = (\rho_0 - \rho) \cdot c^2 \cdot \frac{1}{\rho_0} d\rho$$

as  $\rho = \rho(\rho_0) + c^2(\rho - \rho_0)$   
+ ...

And

$$E_p = \int_{\rho_0}^{\rho} dE = \int_{\rho_0}^{\rho} (\rho_0 - \rho) \cdot c^2 \cdot \frac{1}{\rho_0} d\rho$$

a bit of  
algebra, but  
nothing hard

$$E_p = \frac{1}{2} (\rho - \rho_0)^2 \cdot \frac{c^2}{\rho_0}$$

quadratic  
form?

$$= \frac{1}{2} (\rho - \rho_0)^2 \cdot \frac{1}{\rho_0 c^2}$$

as  $\rho - \rho_0 \approx c^2(\rho - \rho_0)$

$$= \frac{1}{2} (\rho_0 c u)^2 \cdot \frac{1}{\rho_0 c^2}$$

$$= \frac{1}{2} \rho_0 u^2$$

Then the total acoustic energy density  $W$  (kinetic + potential)

$$W = \frac{1}{2} \rho_0 u^2 + \frac{1}{2} \rho_0 u^2$$

$$= \rho_0 u^2$$

Total acoustic energy density

$$W = \rho_0 \cdot u^2$$

while the energy transport (intensity) was

$$J = (\rho - \rho_0) \cdot u = \rho_0 c u^2$$

Thus

$$J = c \cdot W$$

In 3-D we need to rewrite this as

$$W = \frac{1}{2} \rho_0 \left[ \underbrace{(\nabla \phi)^2}_{(u^2 + v^2 + w^2)} + \frac{1}{c^2} \underbrace{\left( \frac{\partial \phi}{\partial t} \right)^2}_{(\rho - \rho_0)^2} \right]$$

and

$$\vec{J} = - \rho_0 \underbrace{\frac{\partial \phi}{\partial t}}_{(\rho - \rho_0)} \cdot \underbrace{\nabla \phi}_{\vec{u}}$$

and

$$\cancel{\frac{\partial W}{\partial t}}$$

which gives

$$\frac{\partial W}{\partial t} = - \nabla \cdot \vec{J}$$

We can then regard  $W$  as a useful quadratic measure of the acoustic amplitude and  $\vec{J}$  as the vector flux of the same quantity

Intensity

$$J = (p - p_0) \cdot u \quad \text{in } \text{Watts/m}^2$$

usually

given in "decibel" (dB) scale

$$120 + 10 \log_{10} (J)$$

where  $J$  in  $\frac{\text{Watts}}{\text{m}^2}$

Blue Whale clicks 188 dB @ 20 Hz

Sperm Whale clicks 230 dB @ 10 Hz

Human hearing threshold depends on frequency

20 Hz to 20,000 Hz

120 dB does injury to ear