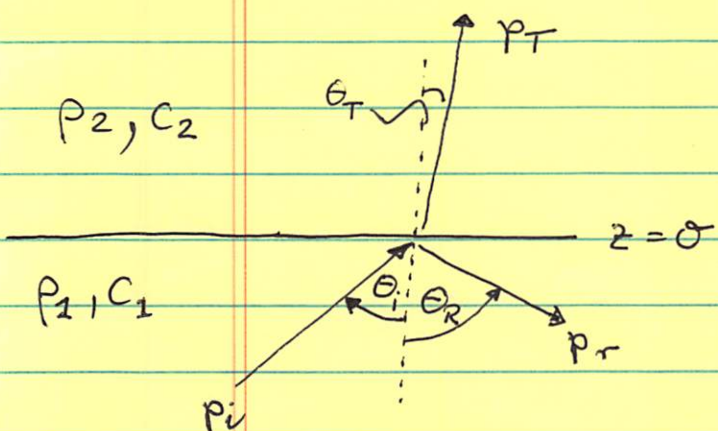
2.5 Scattering at a Discontinuity

Plane wave @ solid boundaries

Plane waves @ boundary of two fluids — air → ocean, ocean → sediment

$\rho_1 \to \rho_2$

$c_1 \to c_2$



$$p_i = a\, e^{i(-\sigma t + kx + m_1 z)}$$

$(+m_1 z) = 0$ @ $z=0$

$(-m_1 z) = 0$ @ $z=0$

$$p_R = R a\, e^{i(-\sigma t + kx - m_1 z)}$$

R reflection coefficient

$$p_T = T a\, e^{i(-\sigma t + kx + m_2 z)}$$

T transmission coefficient

$(+m_2 z) = 0$ @ $z=0$

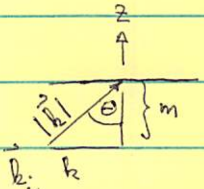
Solution requires pressure and velocity normal to the boundary be continuous at the boundary:

pressure matching $\quad p_i + p_R = p_T \quad$ at $z=0$

velocity matching $\quad \dfrac{1}{\rho_1}\left(\dfrac{\partial p_i}{\partial z} + \dfrac{\partial p_r}{\partial z}\right) = \dfrac{1}{\rho_2}\dfrac{\partial p_T}{\partial z} \quad$ at $z=0$

$$1 + R = T \qquad (1)$$

$$\frac{m_1}{\rho_1}(1-R) = \frac{m_2}{\rho_2} \cdot T \qquad (2)$$



From the dispersion relation $m = \frac{\sigma}{c}\cos\theta$ gives for (2)

$\sigma^2 = c^2(k^2 + l^2 + m^2)$ or $\frac{\sigma}{c} = |\vec{k}|$

$m_1 = \frac{\sigma}{c_1}\cos\theta_i \qquad m_2 = \frac{\sigma}{c_2}\cos\theta_T$

$$\frac{1}{\rho_1 c_1}(1-R)\cos\theta_i = \frac{1}{\rho_2 c_2} T\cos\theta_T \qquad (2)'$$

Treat the two equations for the two unknown R and T

$$R = \rho_2 c_2 \cos\theta_i - \rho_1 c_1 \cos\theta_T \Big/ \rho_2 c_2 \cos\theta_i + \rho_1 c_1 \cos\theta_T$$

$$T = 2\rho_2 c_2 \cos\theta_i \Big/ \quad \text{— " —}$$

$\rho_2 c_2 \ll \rho_1 c_1 \qquad T \to 0 \quad$ and $\quad R \to -1$

→ solid boundary

ocean $\rho_1 \cdot c_1 = 1000 \cdot 1500 \quad kg\, m^{-3}\, m\, s^{-1}$

air $\rho_2 \cdot c_2 = 1 \cdot 340 \quad$ — " —

→ little sound is transmitted from ocean to atmosphere

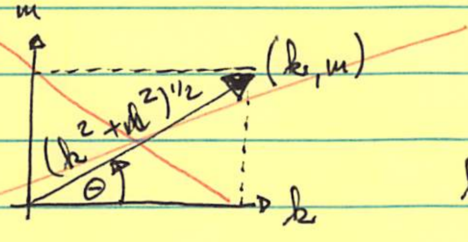
Need to also find the angle of transmitted wave Θ_T

Write down frequency of the wave on both sides of discontinuity

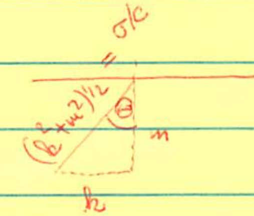
$$\sigma = c_1 (k^2 + m_1^2)^{1/2} = c_2 (k^2 + m_2^2)^{1/2}$$

Snell's Law: $$c_1 \frac{k}{\sin\Theta_i} = c_2 \frac{k}{\sin\Theta_T}$$

because



$$k = (k^2+m^2)^{1/2} \cdot \sin\Theta$$

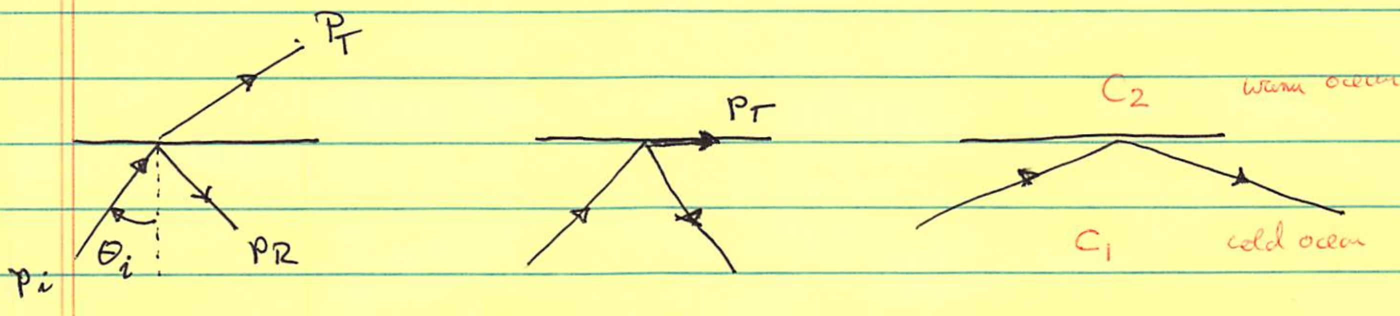


if $c_1 < c_2$ then there exists a critical angle of incidence

~~ocean~~ ~~air~~
cold ocean warm ocean
air ocean

$$\Theta_{ic} = \sin^{-1}(c_1/c_2)$$

beyond which there is total reflection ⟶ internal reflection



$\Theta_i < \Theta_{ic}$ $\Theta_i = \Theta_{ic}$ $\Theta_i > \Theta_{ic}$

$C_{ocean} \propto Temp^2$