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p.54

Ship Waves \rightarrow Waves

We saw that in 1-D $\eta_0(x, t=0) = \delta(x)$, $\eta_{0t}(x, t=0) = 0$ gave solution

$$\eta(x, t) = \operatorname{Re} \left\{ K_1 \frac{P^{1/2}}{x} e^{iP} \right\}$$

with

$$P = g t^2 / 4x$$

In 2-D and radial spreading with $\eta_0(x, y, 0) = \delta(x) \delta(y)$, $\eta_{0t}(x, y, 0) = 0$ we get

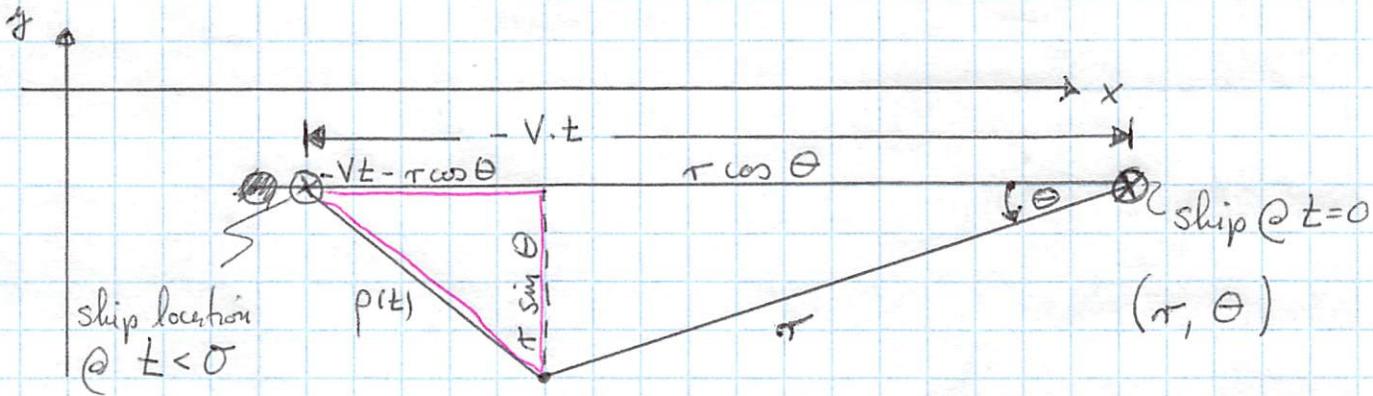
$$\eta(x, y, t) = \operatorname{Re} \left\{ K_2 \frac{P}{r^2} e^{iP} \right\}$$

with

$$P = g t^2 / 4r \quad r = (x^2 + y^2)^{1/2}$$

The (dispersive) oscillatory character e^{iP} is the same, even though the envelope of the oscillations differs from 1-D to 2-D.

A ship is traveling at speed V and represents a traveling δ -function generating waves in deep water:



$$y(r, \theta, t=0) = \int_{-\infty}^0 K_2 \underbrace{\frac{P(t)}{P^2(t)}}_{\text{slow modulation}} e^{i P(t)} dt$$

$$\text{where } P(t) = g t^2 / 4 \rho \quad \text{and} \quad P(t) = \left(r^2 + V^2 t^2 + 2 V t r \cos \theta \right)^{1/2}$$

Solve integral via "stationary phase" ~~assuming $t \rightarrow \infty$ or $P(t) \rightarrow \infty$~~

Points of "stationary phase" are $\frac{dP}{dt} = 0$

(see algebra on p. 40A)

1

$$t^2 + 3t \frac{r \cos \theta}{V} + \frac{2r^2}{V^2} = 0$$

~~cross out~~

quadratic in t

2

$$t_{1/2} = -\frac{3r}{2V} \left[\cos \theta \pm \sqrt{\cos^2 \theta - 8/9} \right]$$

$$P(t) = \frac{gt^2}{4\rho}$$

$$\rho = (\tau^2 + V^2 t^2 + 2Vt r \cos \theta)^{1/2}$$

$$\frac{dP}{dt} = \frac{\frac{2gt}{4} \cdot \rho}{\rho^2} - \frac{\frac{gt^2}{4} \cdot \frac{d\rho}{dt}}{\rho^2}$$

$$= \frac{2gt}{4\rho} - \frac{gt^2}{4\rho^2} \frac{d\rho}{dt}$$

$$\frac{d\rho}{dt} = \frac{1}{2} (2V^2 t + 2V r \cos \theta)$$

$$\frac{2gt}{4\rho} = \frac{gt^2}{4\rho^2} \cdot \frac{1}{2} (2V^2 t + 2V r \cos \theta)$$

$$1 = \frac{4\rho}{2gt} \cdot \frac{gt^2}{4\rho} \cdot \frac{1}{2} (2V^2 t + 2V r \cos \theta) \cdot 2\rho^2$$

$$2\rho^2 = \frac{1}{2} \cdot (2V^2 t^2 + 2V r \cos \theta)$$

$$2\rho^2 = V^2 t^2 + V r t \cos \theta$$

$$2\rho^2 - V^2 t^2 - V r t \cos \theta = 0$$

$$2(\tau^2 + V^2 t^2 + 2V t r \cos \theta) - V t^2 - V t r \cos \theta = 0$$

$$2\tau^2 + V^2 t^2 + 3V t r \cos \theta = 0$$

$$\left| \begin{array}{l} f = \frac{gt^2}{4} \quad g = \rho \\ \frac{d}{dt}(f) = \frac{f \cdot g - f \cdot g'}{g^2} \end{array} \right.$$

Recall that integration ~~is~~ \int_0^∞

$$y(r, \theta, t=0) = \int_{-\infty}^0 \frac{K_2 P(t)}{r^2(t)} e^{i P(t)} dt$$

and we only get contributions

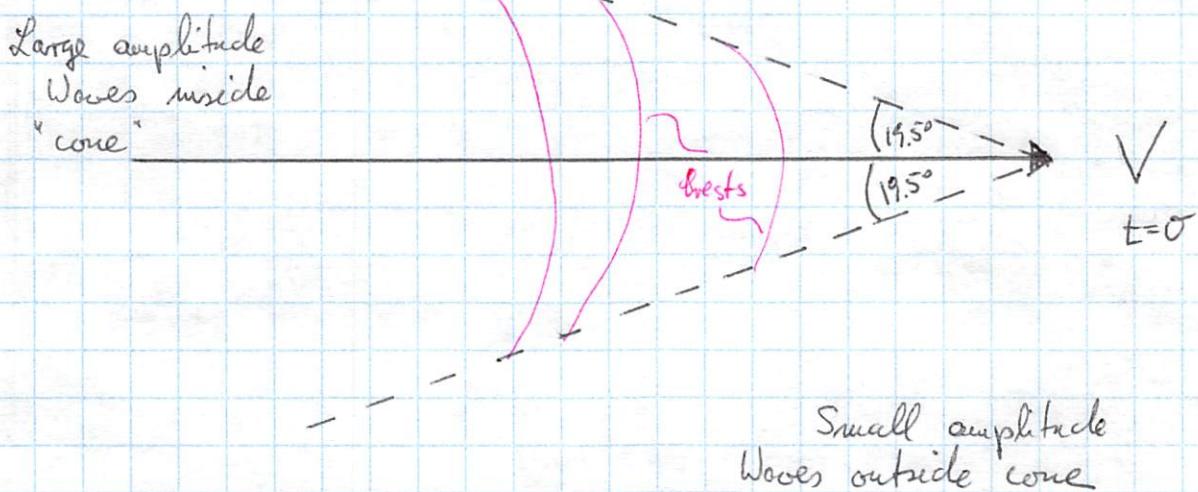
from this integral when $t_{\pm} \in [-\infty, 0]$, that is,

when t_{\pm} are real.

$$\cos^2 \theta > 8/9 \quad \text{or} \quad \theta < 19^\circ 28'$$

Notice that this angle does not depend on the ship speed V , that is, all ships will generate wake waves at the same angle. ~~at~~ [recall ship was a point source $\delta(x)\delta(y)$]

[to generate waves along the line of travel]

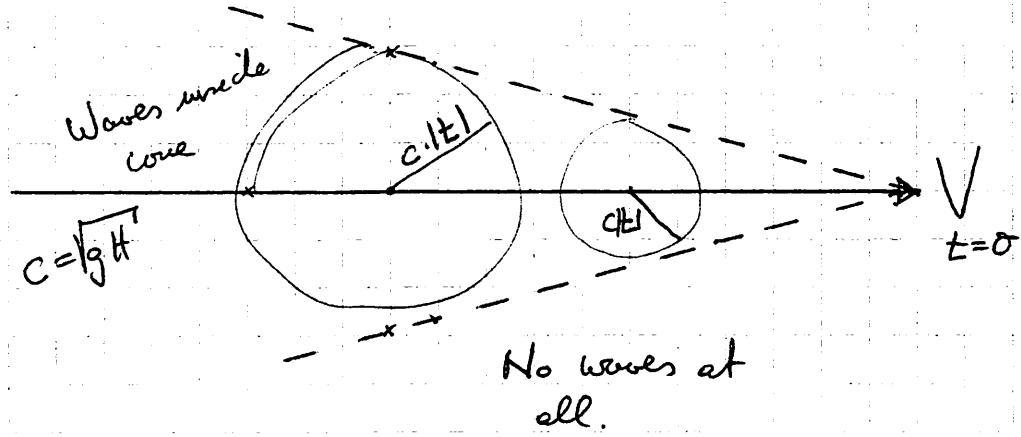


Details of wave shapes are described

by $e^{i P(t_+)} \text{ and } e^{i P(t_-)}$

The independence of ship speed V is surprising,
but recall "dispersive deep water waves
always have some waves that travel faster than V

Non-dispersive shallow water case is different



Waves are confined in cone with $\theta < \sin^{-1} \left(\frac{ct}{Vt} \right)$

which depends on both

the ship's speed and water depth H via $c = \sqrt{gH}$