

~~The normal velocity must vanish on the wall~~



This geometry, law of cosines [$\cos\theta = (a^2 + b^2 - c^2) / 2ab$], and some algebra gives

$$k_r = k_i \left(\frac{1 + aR}{1 - aR} \right) \quad m_r = -m_i \left(\frac{1 + aR}{1 - aR} \right)$$

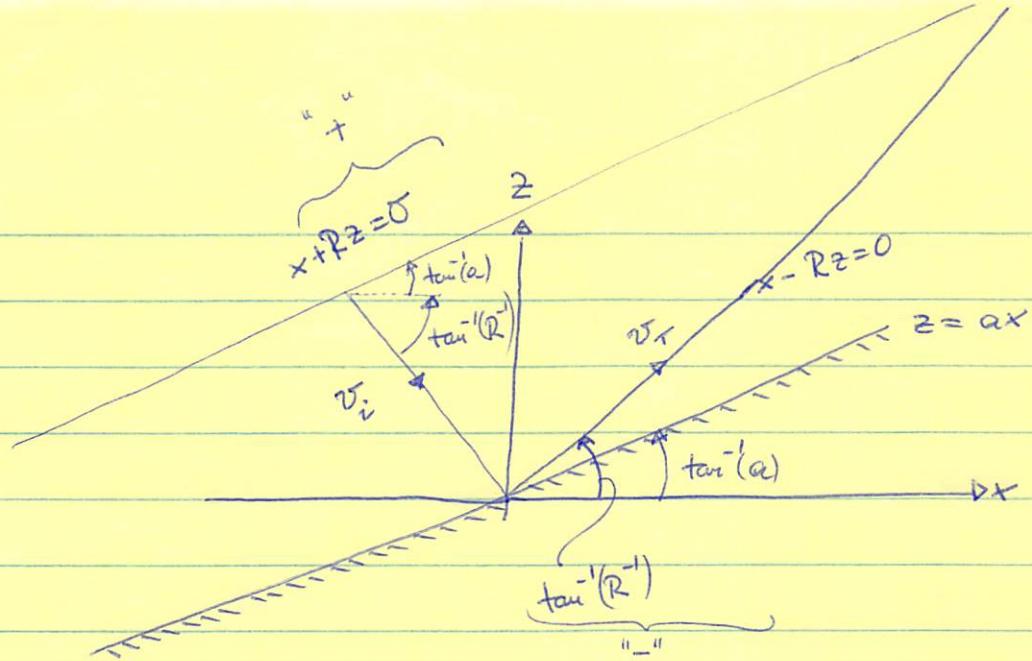
↑ notice sign * ↑

Internal waves at Γ can go only in fixed directions from the vertical φ' ($\pm \tan^{-1} R$ here)

the reflection is NOT normal to the surface, but rather in the direction of the stability vector, that is the vertical

* Notice that there is also the denominator $1 - aR$ that can change sign if $aR > 1$ or $a > R$

↳ there is a critical slope $a_c = R$ that backward reflection occurs



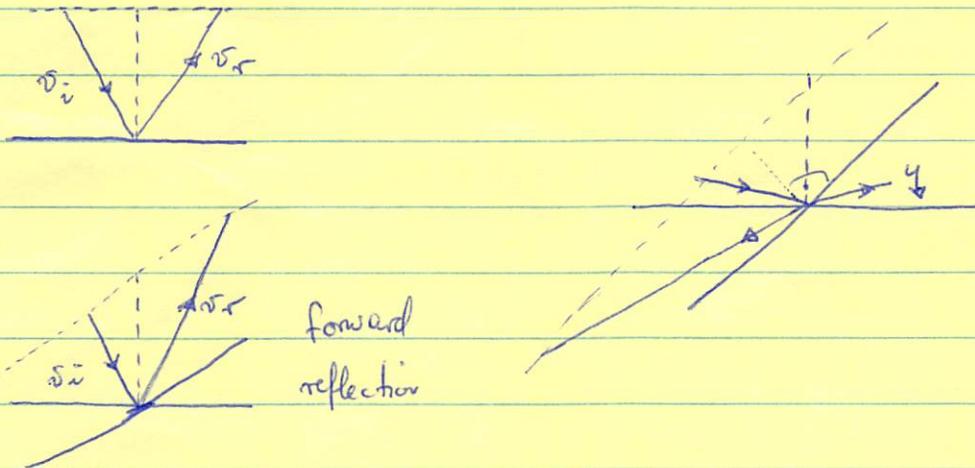
The normal velocity component normal to the solid boundary must vanish

↑

$$v_i \sin [\tan^{-1}(R^{-1}) + \tan^{-1}(\alpha)] = v_r \sin [\tan^{-1}(R^{-1}) - \tan^{-1}(\alpha)]$$

↓

$$v_r = -v_i \frac{1 + \alpha R}{1 - \alpha R}$$



backward
reflection

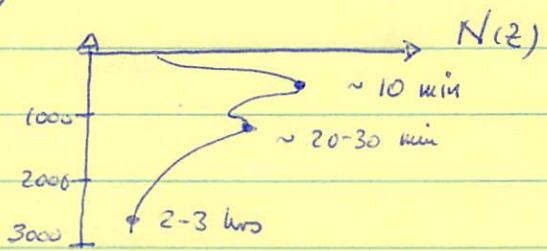


critical forward
reflection $\alpha R \approx 1$

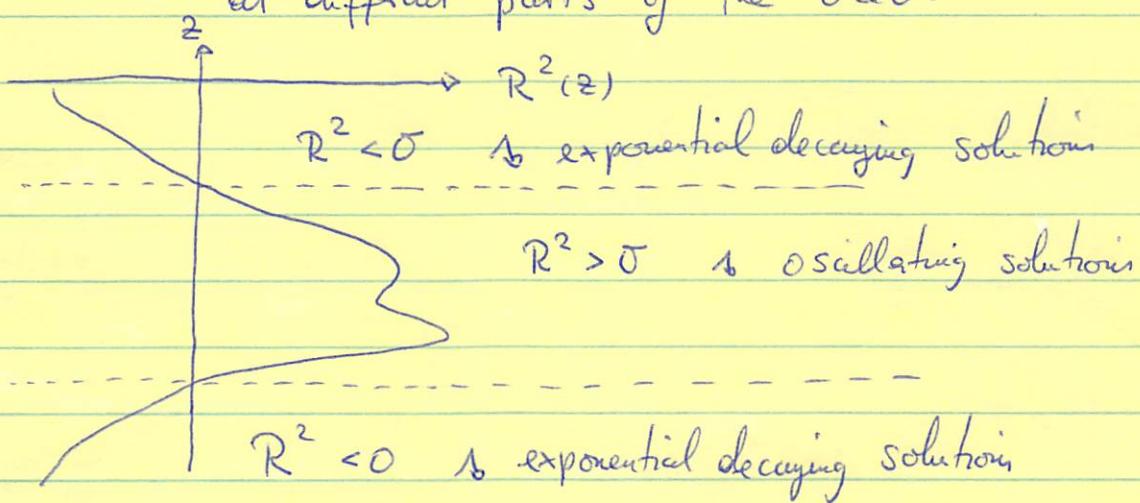
Show example from
Oregon shelf break
Moum + Nash + etc

Variable Buoyancy Frequency $N = N(z)$

$$R^2(z) = \frac{N^2(z) - \sigma^2}{\sigma^2 - f^2}$$



where R^2 may be greater or less than zero at different parts of the ocean



If $R^2(z)$ does not change sign, then all results from $N^2 = \text{const.}$ still apply

numerical
some details of
dispersion and velocity
change

If $R^2(z)$ changes sign, then there are an infinitesimal amount of both evanescent (decaying) and propagating waves present

If $\sigma^2 > f^2$
or $\sigma^2 < f^2$

then there are also two traveling surface waves
two evanescent surface modes.

Shallow Water Equations with Rotation

$$(1) \quad u_t - f v = -\frac{1}{\rho_0} p_x \quad x\text{-momentum}$$

$$(2) \quad v_t + f u = -\frac{1}{\rho_0} p_y \quad y\text{-momentum}$$

$$(3) \quad \sigma = -\frac{1}{\rho_0} p_z - \frac{g \rho}{\rho_0} \quad z\text{-momentum}$$

$$(4) \quad \rho_t + w \rho_0 z = 0 \quad \text{density equation}$$

$$(5) \quad u_x + v_y + w_z = 0 \quad \text{continuity equation}$$

$f = 2\Omega \sin \theta$ where θ is latitude and Ω is magnitude of earth's rotation

Recall that density $\rho^* = \rho_0(z) + \rho(x, y, z, t)$ with $\rho \ll \rho_0$

As before we removed the hydrostatic part and used the Boussinesq Approximation ($\rho_0(z)$ is constant except in density equation)
[p. 51]

Combine vertical momentum with density equation to yield

$$(3') \quad N^2 w = -\frac{1}{\rho_0} p_{zt} \quad N^2 = -\frac{g}{\rho_0} \frac{\partial \rho_0}{\partial z}$$

Let's assume homogeneous fluid, that is, $N^2 = 0$

Homogeneous Fluid (Barotropic Motions)

Hydrostatic vertical momentum

$$p_z = -g \rho_0 \quad (\rho \text{ is total pressure not perturbation})$$

integrate

$$p(z=y) - p(z) = -g \rho_0 (y - z)$$

or

$$\begin{aligned} p(z) &= \underbrace{p(z=y)}_{p_{\text{atmosphere}} = 0} + g \rho (y - z) \\ &= +g \rho (y - z) \end{aligned}$$

$$\downarrow u_t - fv = -g \gamma_x$$

$$v_t + fu = +g \gamma_y$$

$$u_x + v_y + w_z = 0$$

integrate continuity goes from $z=-D$ to $z=y$

$$\int_{-D}^y (u_x + v_y) dz + w \Big|_{z=y} - w \Big|_{z=-D} = 0$$

BC are

$$w = \frac{Dy}{Dt} \quad \text{at } z=y \quad \text{and} \quad w = -uD_x - vD_y$$

at $z=-D$

$$\eta_t + [u(\eta + D)]_x + [v(\eta + D)]_y = 0$$

For $\eta \ll D$ ~~$\eta_t + (uD)_x + (vD)_y = 0$~~

These are the so-called shallow water equations with rotation.
(non-rotating version was derived in surface gravity waves)

Let's include stratification in constant depth ($D = \text{const}$) ocean

$$u = U(x, y, t) \cdot F(z)$$

$$v = V(x, y, t) \cdot F(z)$$

$$w = W(x, y, t) \cdot G(z)$$

$$p = P(x, y, t) \cdot H(z)$$

$$(1) \quad (U_z - fV) \cdot F = -\frac{1}{\rho_0} P_x H \quad \Rightarrow u_z - fv = -\frac{1}{\rho_0} p_x$$

$$(2) \quad (V_z + fU) \cdot F = -\frac{1}{\rho_0} P_y H \quad v_z + fu = -\frac{1}{\rho_0} p_y$$

$$(3) \quad N^2 W G = -\frac{1}{\rho_0} P_z H_z \quad N^2 w = -\frac{1}{\rho_0} p_z t$$

$$(4) \quad (U_x + V_y) F + W G_z = 0 \quad u_x + v_y + w_z = 0$$