3. Coriolis in Local Co-ordinates

We choose to observe in a local, rotating co-ordinate system fixed to the surface of the earth with x oriented to the East, y oriented to the north, and z oriented in the direction of the effective gravity, that is, $\vec{g}_{eff} = (0, 0, g)$.

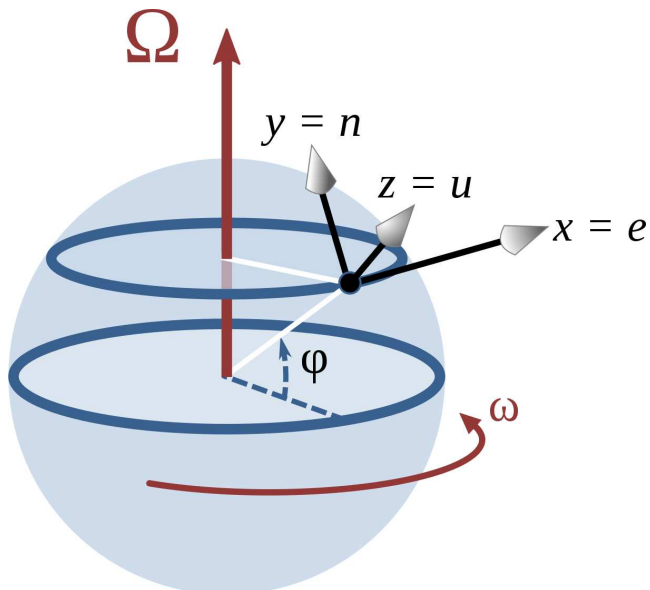

FIG. 2. *Unit vectors east ("e"), north ("n"), and up ("u") at the surface of a rotating earth. [Adapted from https : //en.wikipedia.org/wiki/Coriolis_force]*

In this co-ordinate system, the earth's rotational vector becomes, with ϕ the latitude

$$\vec{\Omega} = \begin{pmatrix} 0 \\ \Omega_y \\ \Omega_z \end{pmatrix} = \Omega \cdot \begin{pmatrix} 0 \\ cos(\phi) \\ sin(\phi) \end{pmatrix} \tag{16}$$

The Coriolis acceleration then becomes

$$2 \cdot \vec{\Omega} \times \vec{u} = \begin{pmatrix} 0 \\ 2\,cos(\phi) \\ 2\,sin(\phi) \end{pmatrix} \times \begin{pmatrix} u_e \\ v_n \\ w_u \end{pmatrix} = \begin{pmatrix} w_u \cdot 2\,cos(\phi) \;-\; v_n \cdot 2\,sin\phi \\ u_e \cdot 2\,sin(\phi) \\ -u_e \cdot 2\,cos(\phi) \end{pmatrix} \approx \begin{pmatrix} -v_n \cdot 2\,sin(\phi) \\ u_e \cdot 2\,sin(\phi) \\ -u_e \cdot 2\,cos(\phi) \end{pmatrix} \tag{17}$$

The last approximation uses $w_u \ll v_n$ and subsequently we write $f = 2sin(\phi)$ as the Coriolis parameter that varies with latitude ϕ.