# Inverse Ray Tracing Model

$$\frac{\partial}{\partial t}\left[p_{xx} + p_{yy} + \left(\frac{f_0}{N}\right)^2 p_{zz}\right] + \beta p_x = 0$$

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x-mom $$\frac{\partial u}{\partial t} - f_0 v - \beta_0 g v = -\frac{1}{\rho_0} p_x + \gamma u_{xx}$$

y-mom $$\frac{\partial v}{\partial t} + f_0 u - \beta_0 y u = -\frac{1}{\rho_0} p_x + \gamma v_{yy}$$

z-mom $$0 = p_z - \rho' g$$

Density $$\rho'_t - \rho_0 N^2 w/g = 0$$

Scaling and Taylor series to find equation for $u, v$

↓ Combine with Continuity

contin. $$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0$$

↓ Quasigeostrophic Vorticity Equation

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Boundary Conditions

$$\frac{\partial}{\partial t} p_z = 0 \ @ \ z = 0$$ ....................................... Rigid lid

$$\frac{\partial}{\partial t} p_z = \frac{N^2}{f_0}\boxed{\left(p_x h_y - p_y h_x\right)} \ @ \ z = -h$$ .......... No normal flow through bottom

bottom slopes in (x,y)

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Input solution for pressure as,

$$p = A(z) e^{i(kx + ly + \omega t)}$$

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$A_{zz} - \lambda_v A = 0$

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Coupled dispersion relation,

$$\lambda_v^2 = \left(k^2 + l^2 + \frac{\beta k}{\omega}\right)\left(\frac{N}{f_0}\right)^2,$$

$$\lambda_v \tanh(\lambda_v h) = \frac{N^2}{\omega f_0}(kh_y - lh_x)$$

$$\omega = \frac{\beta k}{\left(\frac{f_0 \lambda_v}{N}\right)^2 - (k^2 + l^2)}$$

$$\omega = \frac{N^2}{f_0}\left(\frac{kh_y - lh_x}{\lambda_v \tanh(\lambda_v h)}\right)$$

Need to solve this ODE for A(z) subject to boundary conditions

Try $A(z) = A_0 * \cosh(\lambda_v h)$

My thought?

If……… $\left(\frac{f_0 \lambda_v}{N}\right)^2 > k_H^2$

Then………… $k > 0$

If……… $\left(\frac{f_0 \lambda_v}{N}\right)^2 < k_H^2$

Then………… $k < 0$