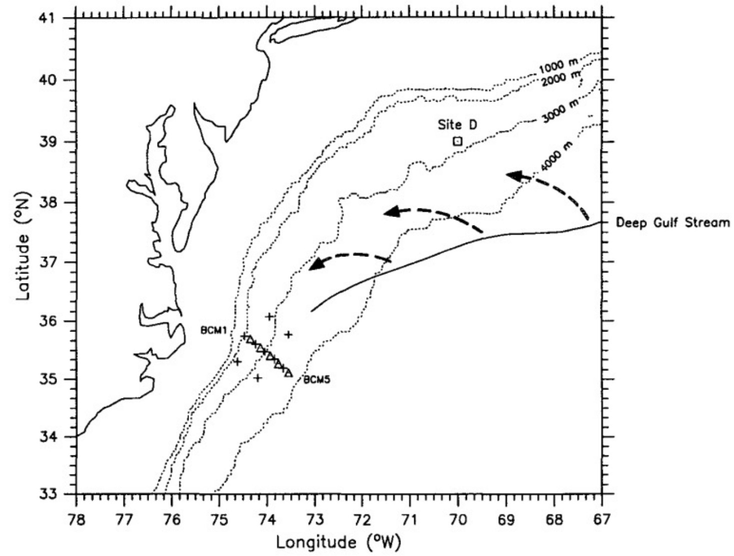
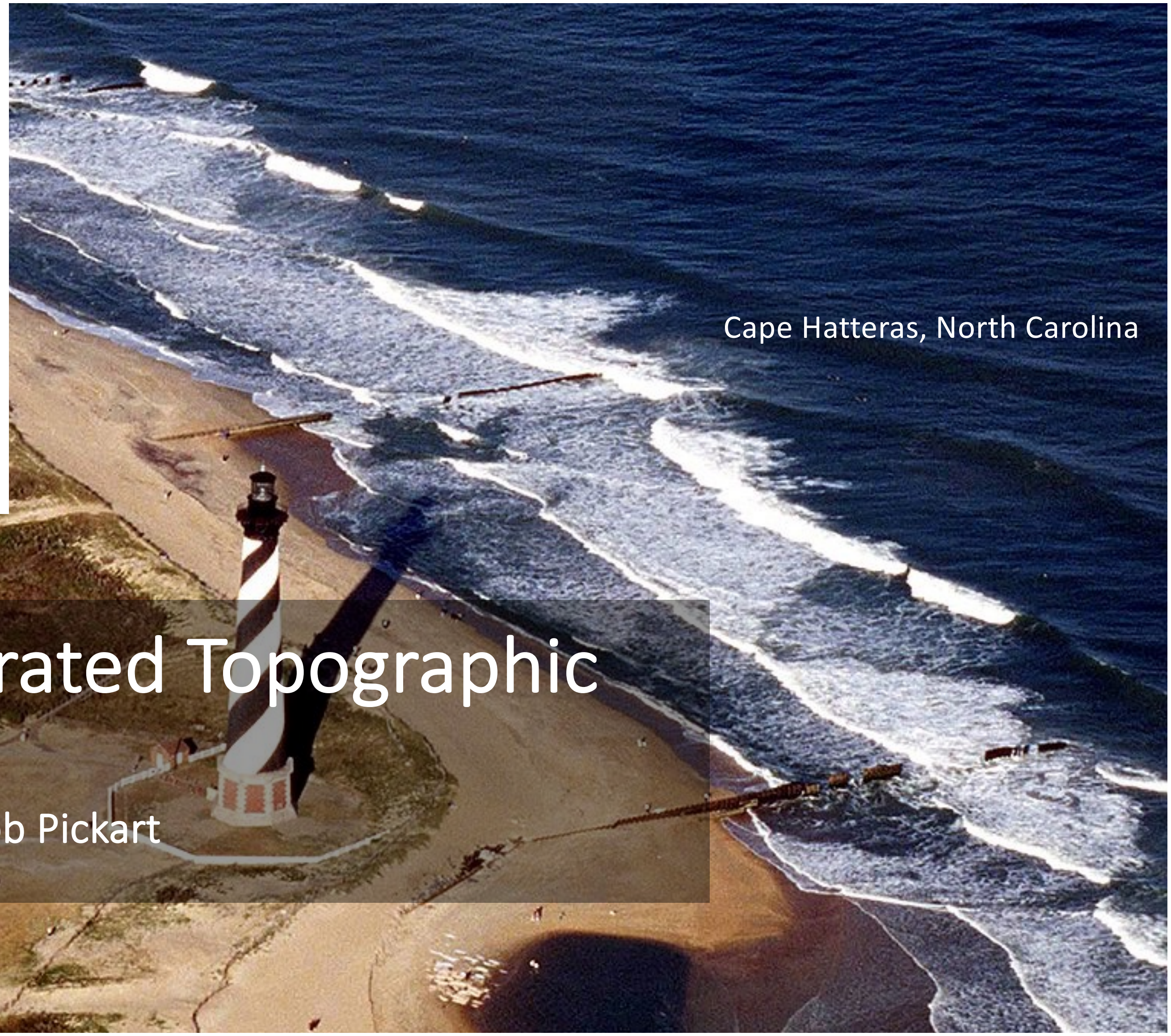
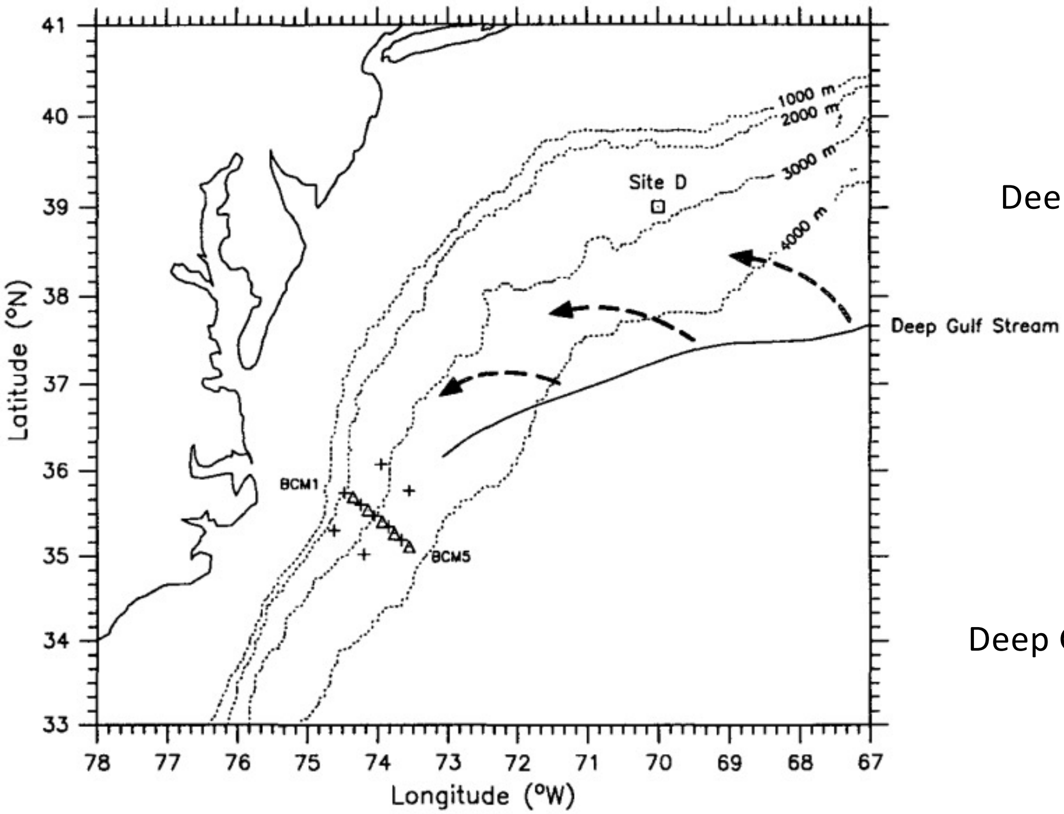




Gulf Stream-Generated Topographic Rossby Waves

Bob Pickart

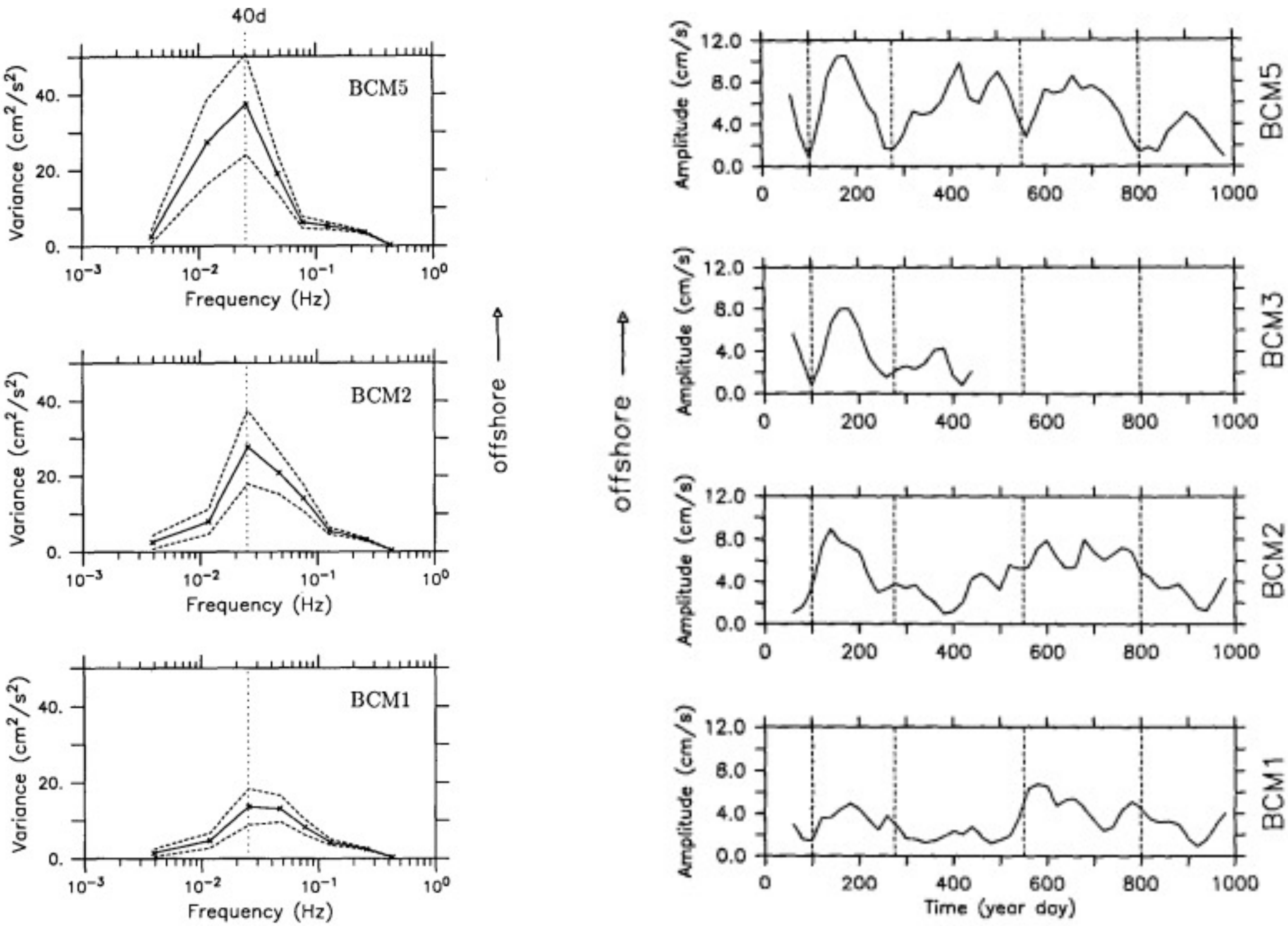


Deep mesoscale variability due to topographic Rossby waves

Periods of 10-60 days

Deep Gulf stream surface meanders do not extend to the bottom

Gulf Stream is indicated as the obvious source of the topographic waves, the precise generation mechanism is still unclear.



Preliminary observations

Inverse Ray Tracing Model

$$\frac{\partial}{\partial t}\left[p_{xx} + p_{yy} + \left(\frac{f_0}{N}\right)^2 p_{zz}\right] + \beta p_x = 0$$

Inverse Ray Tracing Model

x-mom $$\frac{\partial u}{\partial t} - f_0 v - \beta_0 g v = -\frac{1}{\rho_0} p_x + \gamma u_{xx}$$

y-mom $$\frac{\partial v}{\partial t} + f_0 u - \beta_0 y u = -\frac{1}{\rho_0} p_x + \gamma v_{yy}$$

z-mom $$0 = p_z - \rho' g$$

Scaling and Taylor series to find equation for u, v

Combine with Continuity

contin. $$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0$$

Quasigeostrophic Vorticity Equation

$$\frac{\partial}{\partial t}\left[p_{xx} + p_{yy} + \left(\frac{f_0}{N}\right)^2 p_{zz}\right] + \beta p_x = 0$$

Inverse Ray Tracing Model

Quasigeostrophic Vorticity Equation

$$\frac{\partial}{\partial t}\left[p_{xx} + p_{yy} + \left(\frac{f_0}{N}\right)^2 p_{zz}\right] + \beta p_x = 0$$

Boundary Conditions

$$\frac{\partial}{\partial t} p_z = 0 \text{ @ } z = 0$$ .... Rigid lid

$$\frac{\partial}{\partial t} p_z = \frac{N^2}{f_0}\left(p_x h_y - p_y h_x\right) \text{ @ } z = -h$$ .... No normal flow through bottom

Inverse Ray Tracing Model

Quasigeostrophic Vorticity Equation

$$\frac{\partial}{\partial t}\left[p_{xx} + p_{yy} + \left(\frac{f_0}{N}\right)^2 p_{zz}\right] + \beta p_x = 0$$

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Input solution for pressure as,

$$p = A(z)e^{i(kx+ly+\omega t)}$$

Inverse Ray Tracing Model

Quasigeostrophic Vorticity Equation

$$\frac{\partial}{\partial t}\left[p_{xx} + p_{yy} + \left(\frac{f_0}{N}\right)^2 p_{zz}\right] + \beta p_x = 0$$

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Input solution for pressure as,

$$p = A(z)e^{i(kx+ly+\omega t)}$$

Coupled dispersion relation,

$$\lambda_v^2 = \left(k^2 + l^2 + \frac{\beta k}{\omega}\right)\left(\frac{N}{f_0}\right)^2,$$

$$\lambda_v \tanh(\lambda_v h) = \frac{N^2}{\omega f_0}(kh_y - lh_x)$$

$$\omega = \frac{\beta k}{\left(\frac{f_0 \lambda_v}{N}\right)^2 - (k^2 + l^2)}$$

$$\omega = \frac{N^2}{f_0}\left(\frac{kh_y - lh_x}{\lambda_v \tanh(\lambda_v h)}\right)$$

My thought?

If......... $\left(\frac{f_0 \lambda_v}{N}\right)^2 > k_H^2$

Then............. $k > 0$

If......... $\left(\frac{f_0 \lambda_v}{N}\right)^2 < k_H^2$

Then............. $k < 0$

Inverse Ray Tracing Model

Quasigeostrophic Vorticity Equation

$$\frac{\partial}{\partial t}\left[p_{xx} + p_{yy} + \left(\frac{f_0}{N}\right)^2 p_{zz}\right] + \beta p_x = 0$$

Boundary Conditions

$$\frac{\partial}{\partial t} p_z = 0 \text{ @ } z = 0$$

$$\frac{\partial}{\partial t} p_z = \frac{N^2}{f_0}\left(p_x h_y - p_y h_x\right) \text{ @ } z = -h$$

Input solution for pressure as,

$$p = A(z)e^{i(kx+ly+\omega t)}$$

Coupled dispersion relation,

$$\omega = \frac{\beta k}{\left(\frac{f_0 \lambda_v}{N}\right)^2 - (k^2 + l^2)}$$

$$\omega = \frac{N^2}{f_0}\left(\frac{k h_y - l h_x}{\lambda_v \tanh(\lambda_v h)}\right)$$

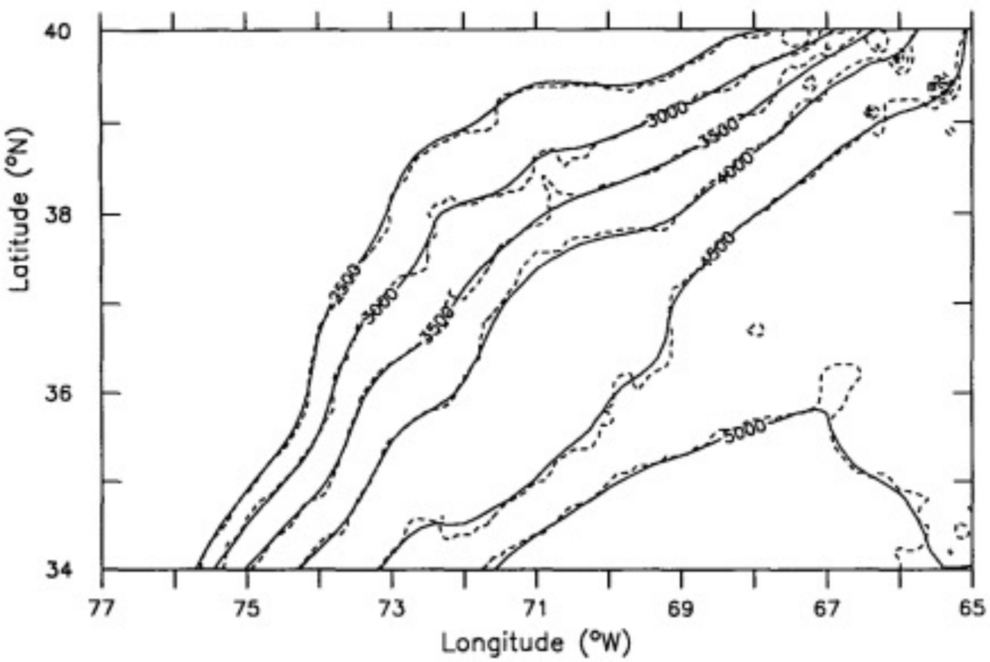
Equations governing wave and wavenumber,

$$\frac{Dx}{Dt} = \frac{\partial \omega}{\partial k} = c_g$$

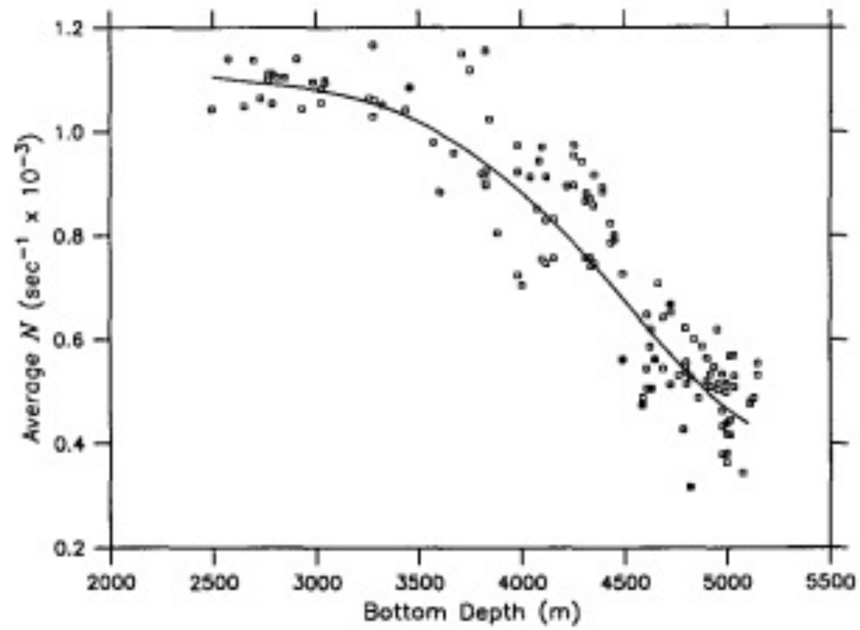
$$\frac{Dk}{Dt} = \sum -\frac{\partial \omega}{\partial \gamma_i} \nabla \gamma_i$$

γ_i are the environmental factors. It contains h (water depth), ∇h (bottom slope), and N (Brunt-Vaisala frequency)

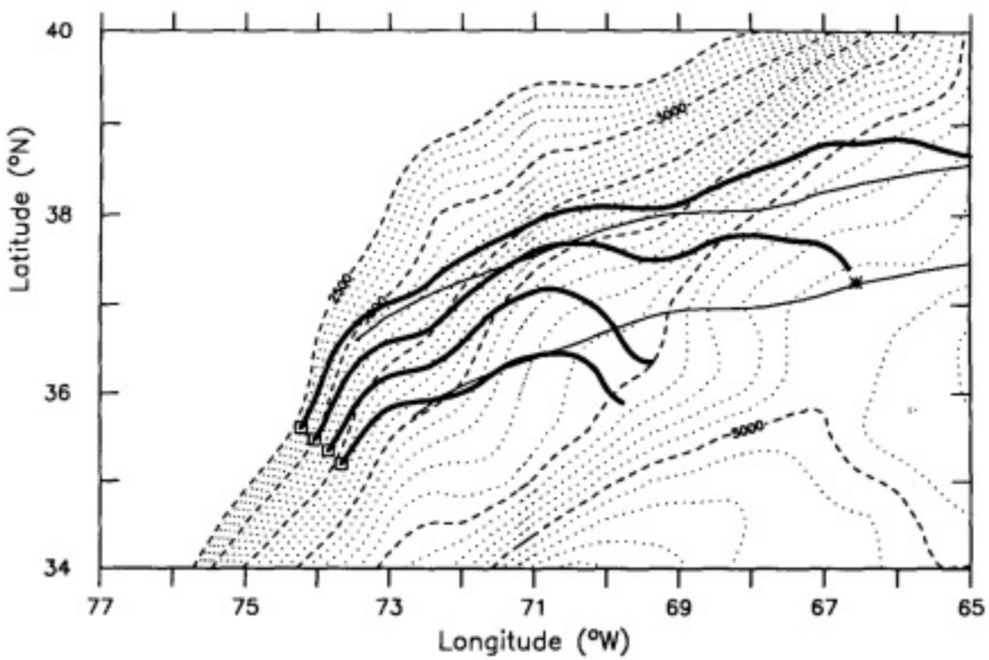
Simplified System



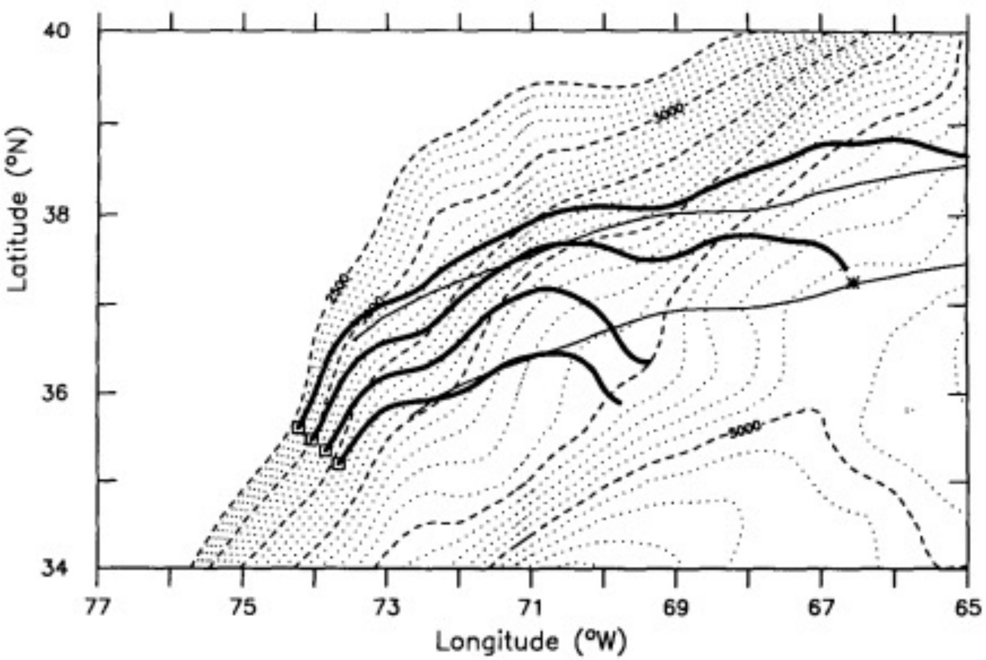
Spline fit to a filtered bathymetry

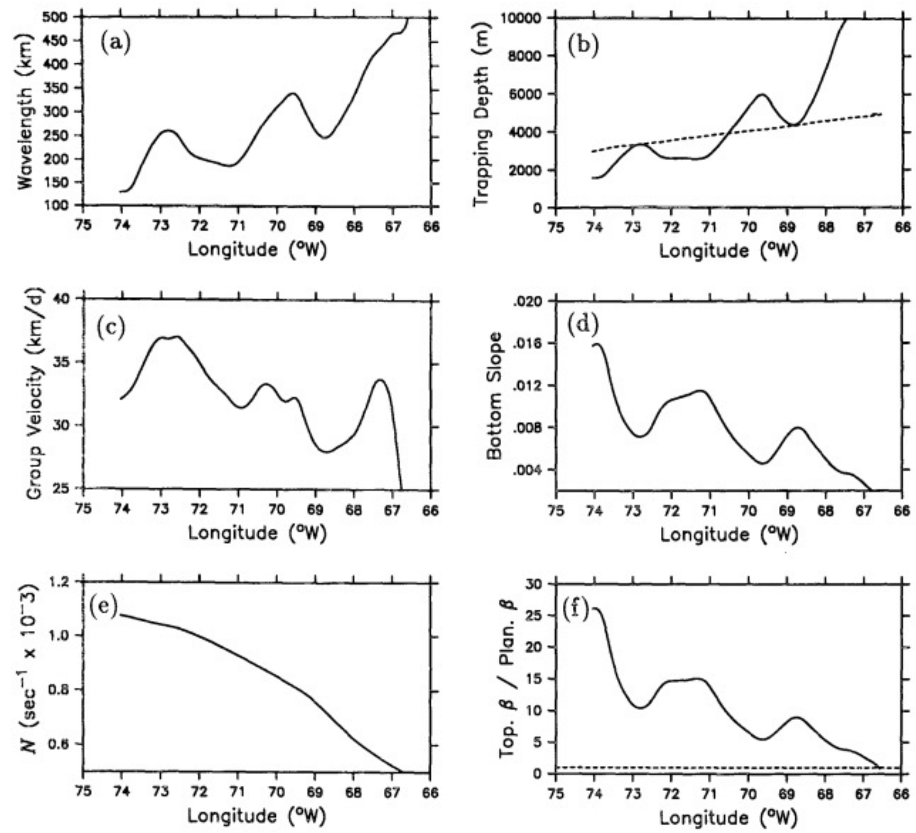


Spline fit to buoyancy frequency versus depth



40
38
36
34
Latitude (°N)
77
75
73
71
69
67
65
Longitude (°W)
3000
2500
5000





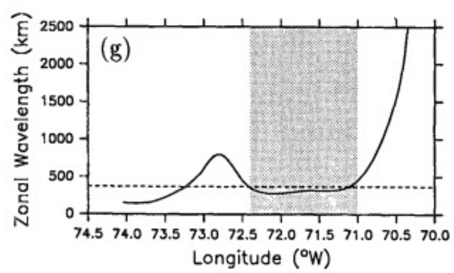
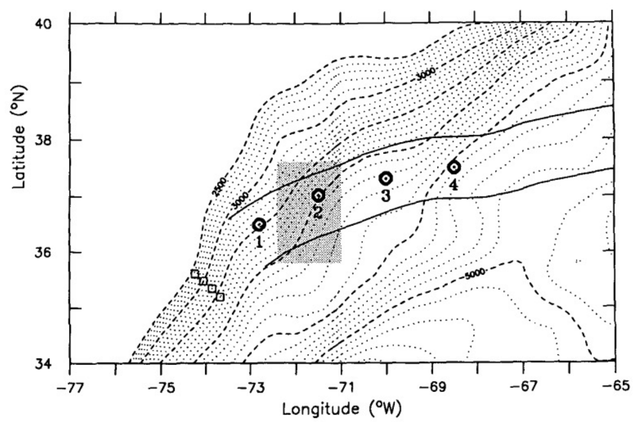
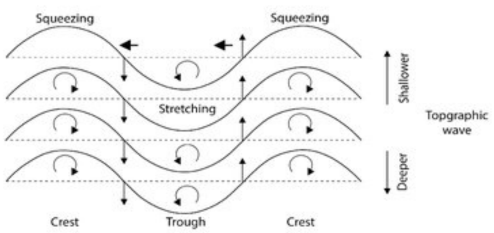
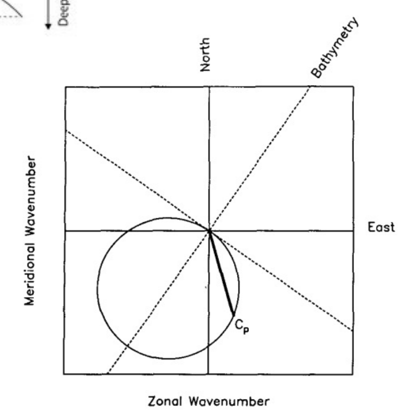


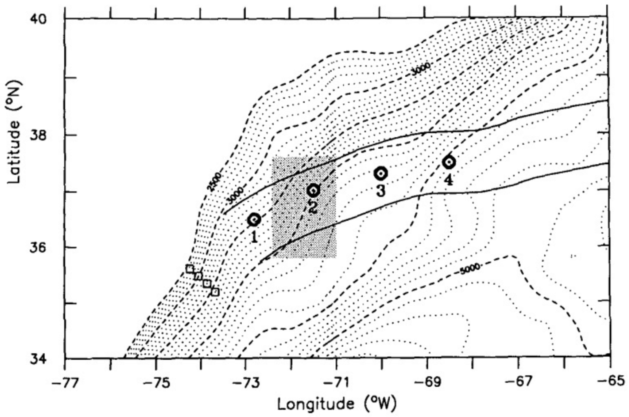
FIG. 8. Features of the 40-day Rossby wave along the ray path denoted by the asterisk in Fig. 7 (note: choosing any of the rays in Fig. 7 gives comparable results): (a) wavelength $2\pi/|\mathbf{k}|$; (b) vertical trapping scale $1/\lambda_v$ (solid line), bottom depth (dashed line); (c) group velocity; (d) bottom slope; (e) Brunt–Väisälä frequency N; (f) ratio of topographic β to planetary β (solid line), the dashed line is the value 1; and (g) zonal wavelength $2\pi/k$ (solid line). The dashed line is the wavelength of the most frequently occurring meander in the Gulf Stream. The shaded band marks the region over which the two curves match.

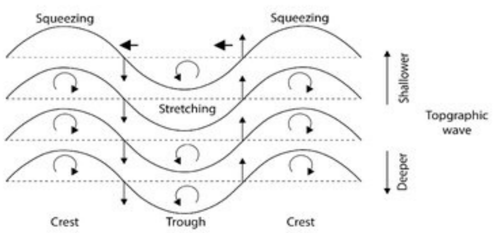


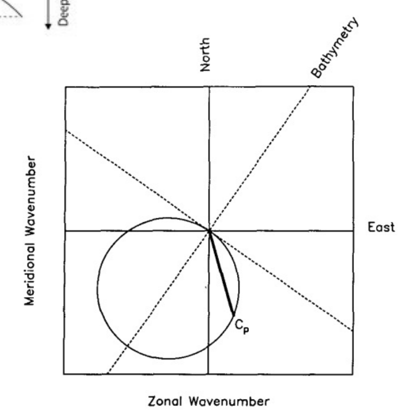


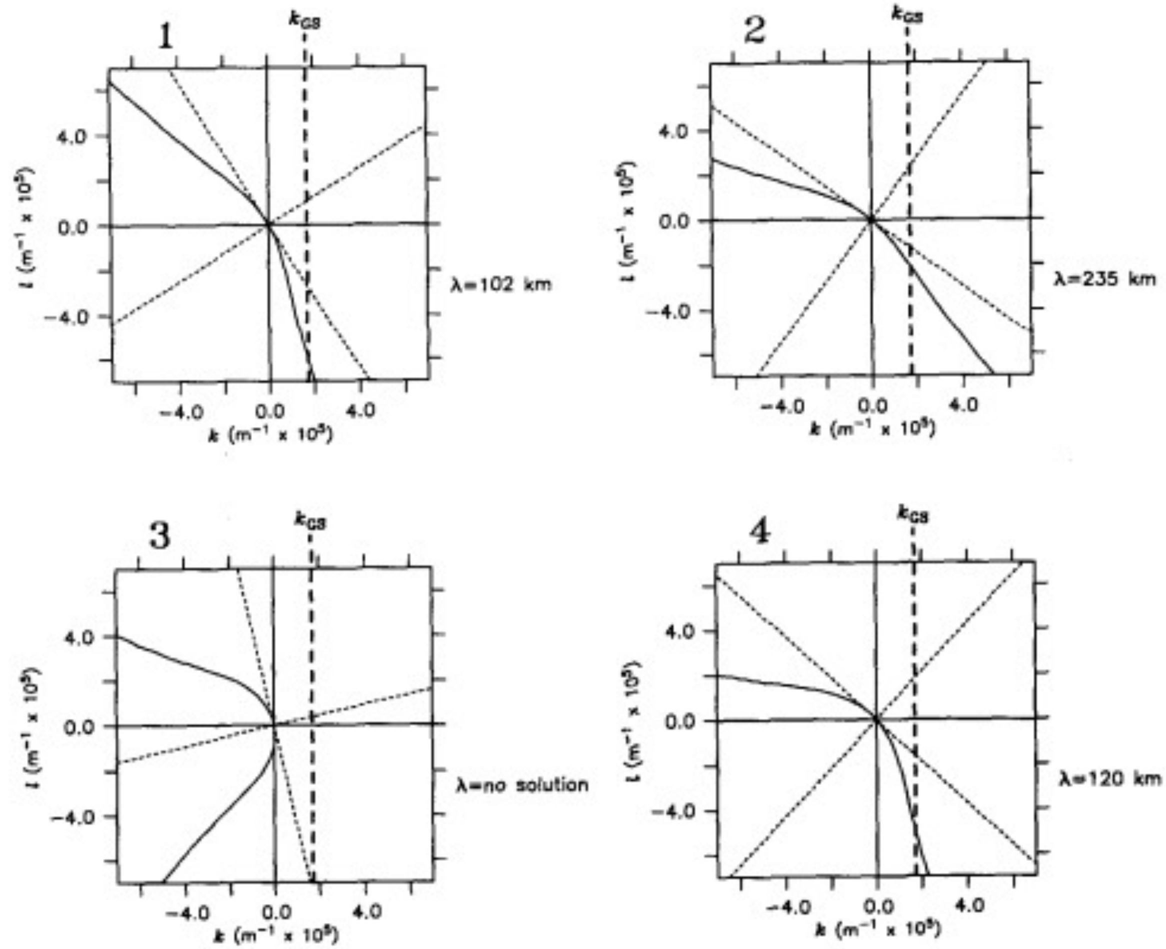


Dispersion relationships at four locations

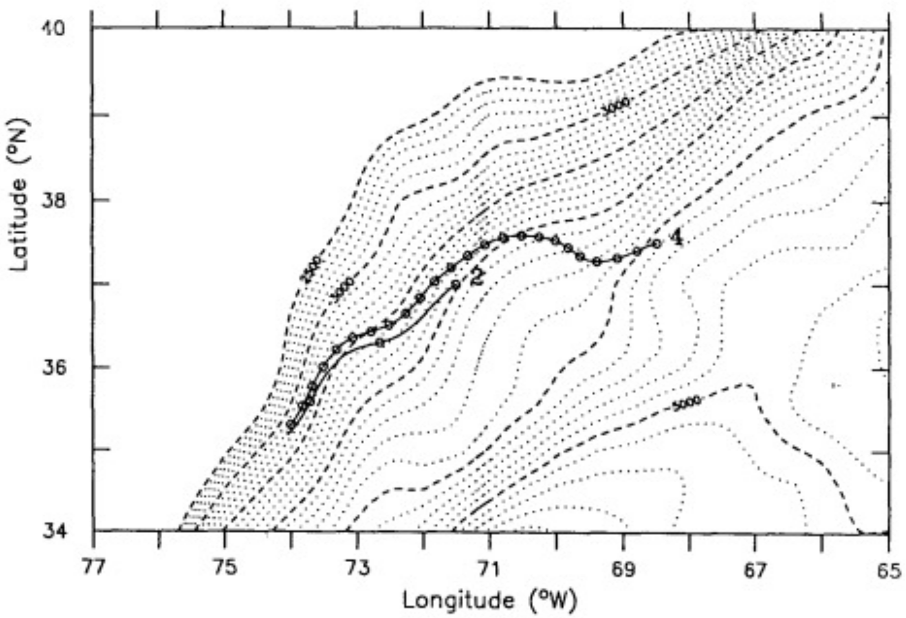




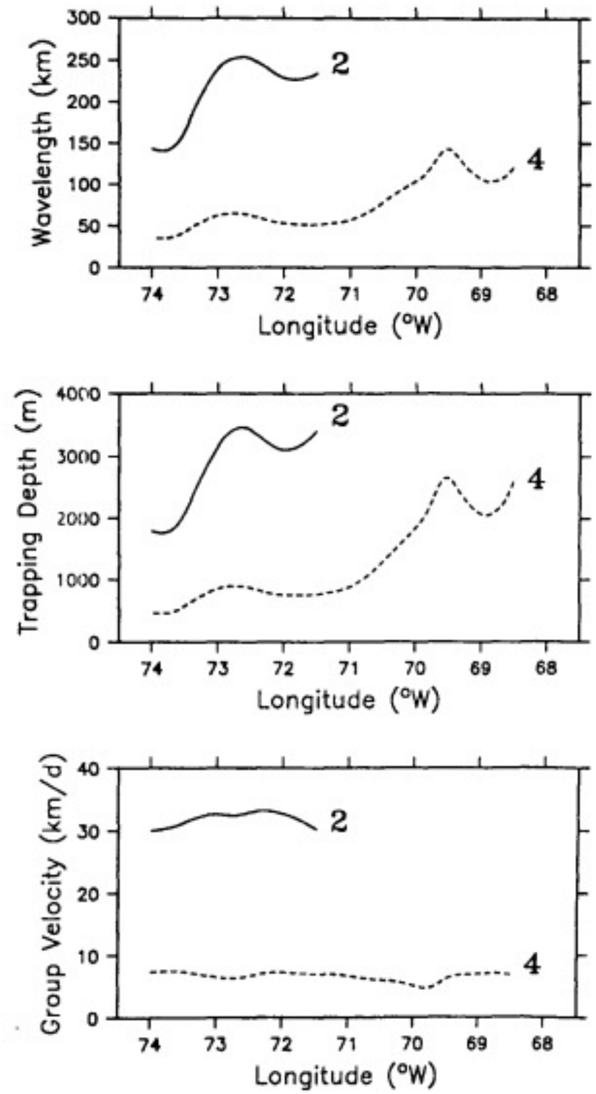


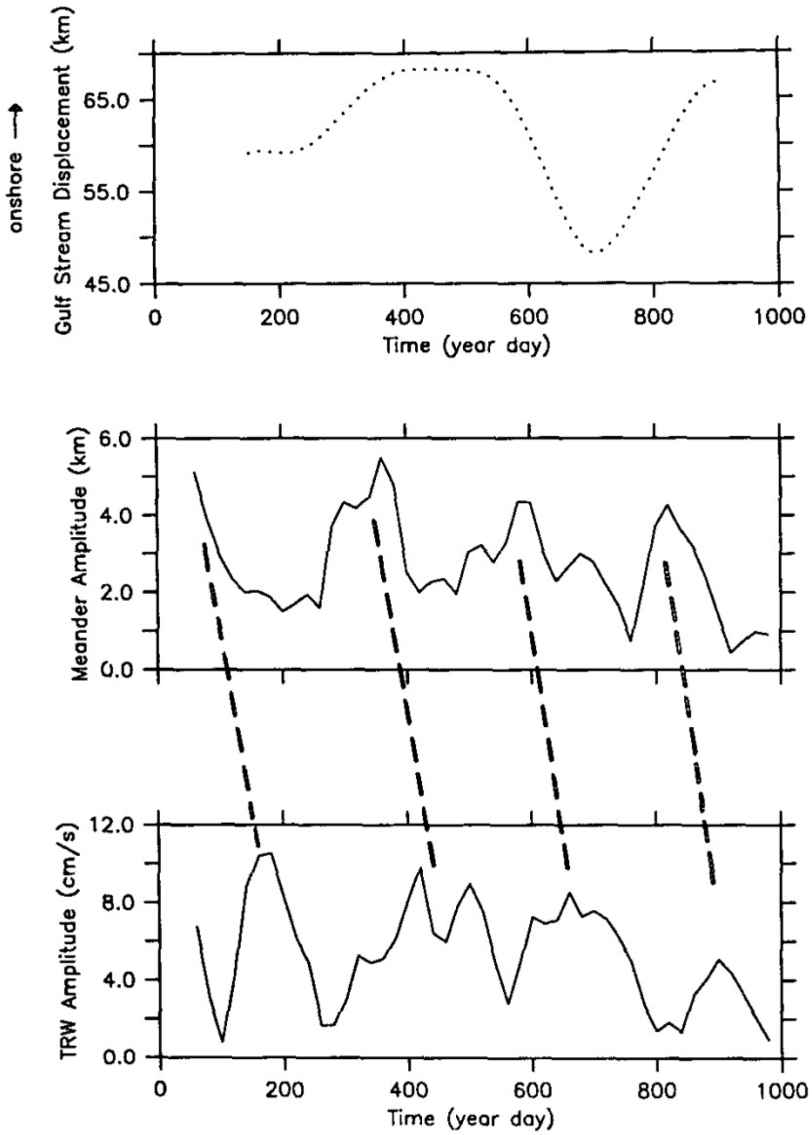


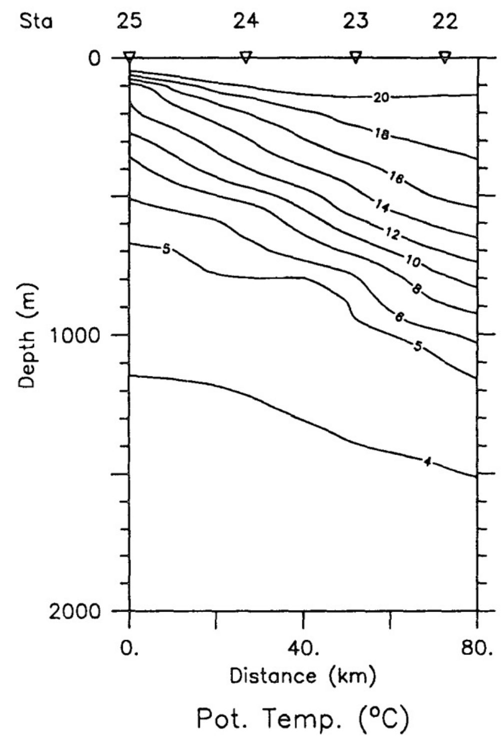
Dispersion relationships at four locations

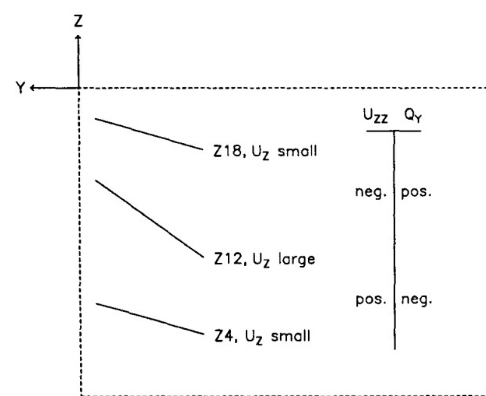


Forward solution of the model at two different locations









Potential Vorticity

$$Q_y \sim \beta - \frac{f_0^2}{N^2} u_{zz}$$

Conclusions

40-day topographical Rossby waves occur in regular bursts off Cape Hatteras

Provided evidence for the Gulf Stream energy radiation mechanism

Bursts of these meandors are associated with large-scale transitions in the position of the Gulf Stream

Unanswered questions: What triggers the bursts and what determines their period?