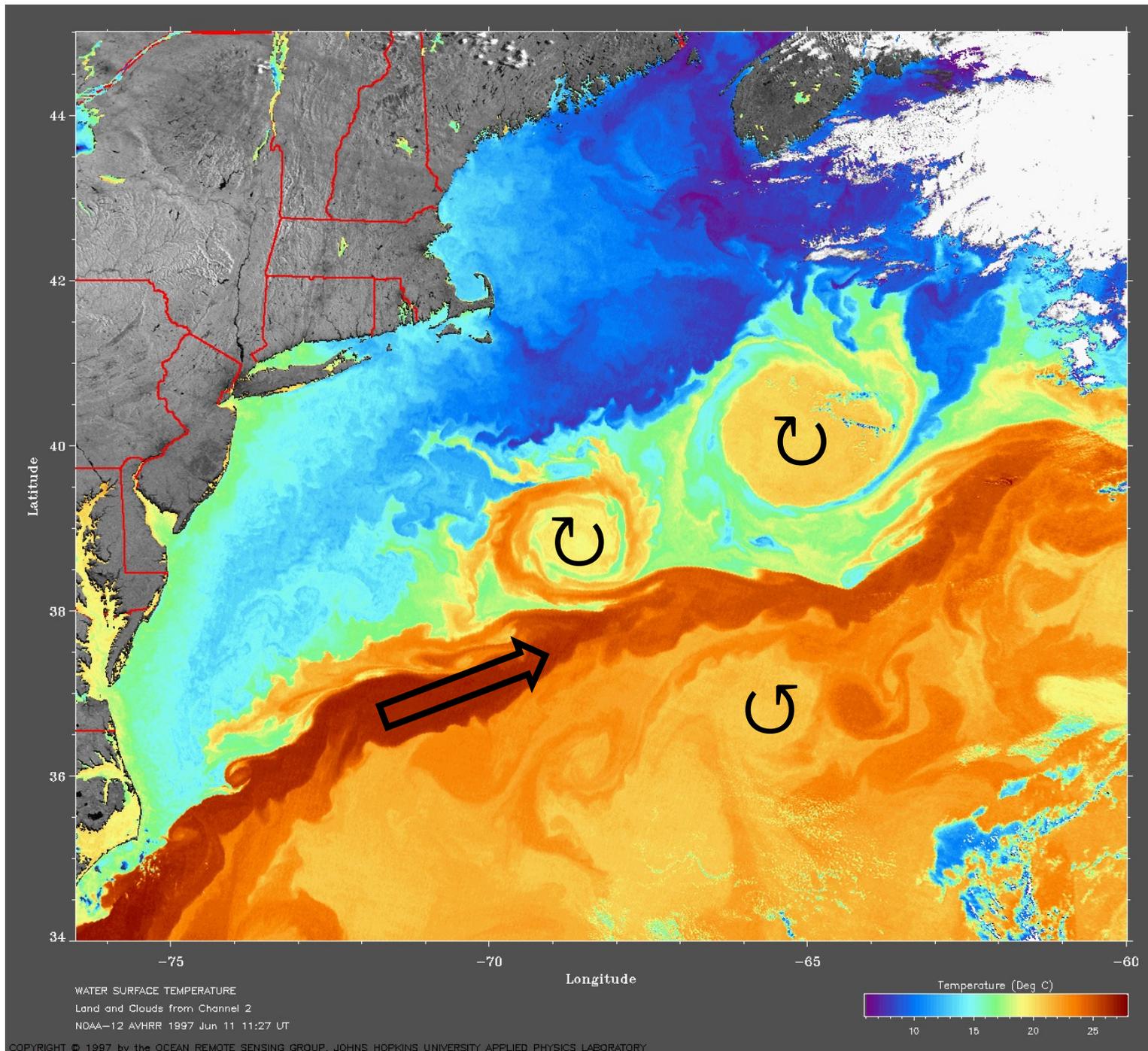


# Sea Surface Temperature: Gulf Stream Eddies 1997



↺ cyclonic

↻ anti-cyclonic

↕  
 $L \sim 200 \text{ km}$

$L \sim 200 \text{ km}$

$U \sim 1 \text{ m/s}$

$L/U \sim 50 \text{ hrs}$

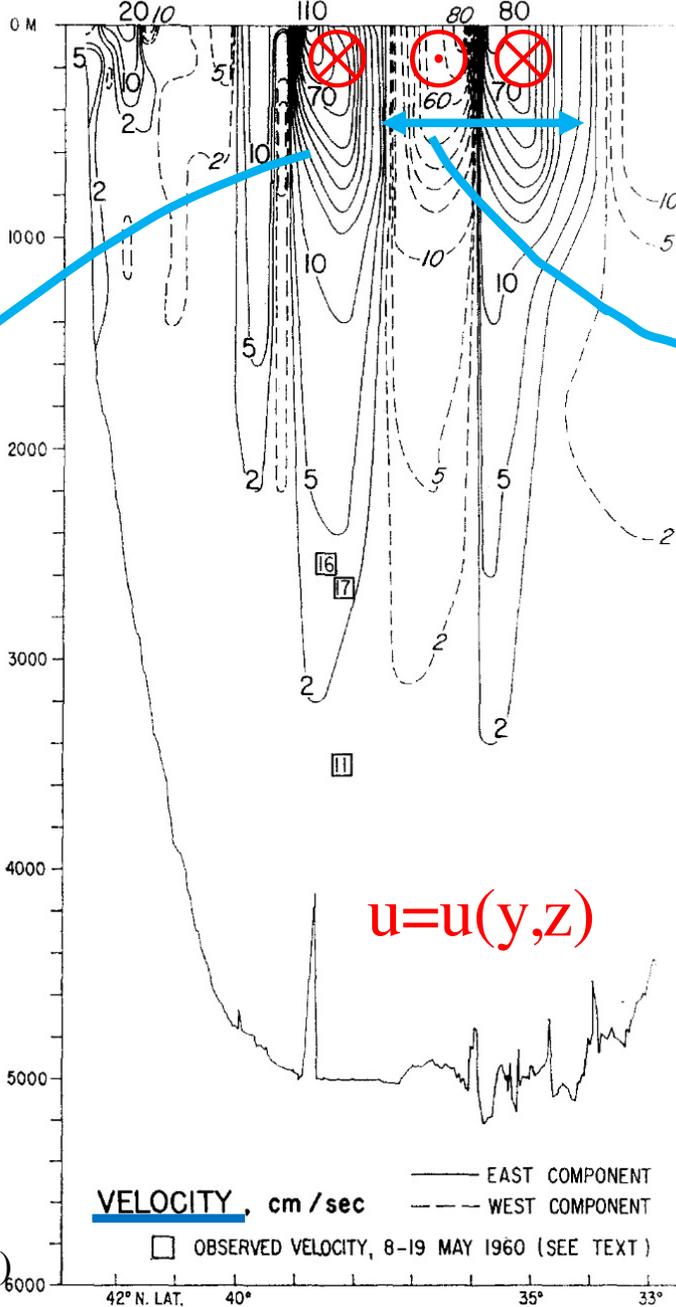
$Ro = (U/L) / \Omega$   
 $\sim 0.1$

GFD

$L \sim 100 \text{ km}$



STATION 5907 5910 5915 5920 5926



$H \sim 1 \text{ km}$



Cyclonic eddy

or

Gulf Stream meander

Gulf Stream

Here  $L$  is oceanic mesoscale  $\sim 100 \text{ km}$

$U \sim 0.5 \text{ m/s}$

$\partial U / \partial z \sim U / H \sim 0.5 \times 10^{-3} \text{ s}^{-1}$

0.5 hrs

$\partial U / \partial y \sim U / L \sim 0.5 \times 10^{-5} \text{ s}^{-1}$

55 hrs

Fuglister (1963)

**TABLE 1-1 LENGTH AND VELOCITY SCALES OF MOTIONS IN WHICH ROTATION EFFECTS ARE IMPORTANT**

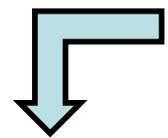
$L = 1 \text{ m}$	$U \leq 0.012 \text{ mm/s}$
$L = 10 \text{ m}$	$U \leq 0.12 \text{ mm/s}$
$L = 100 \text{ m}$	$U \leq 1.2 \text{ mm/s}$
$L = 1 \text{ km}$	$U \leq 1.2 \text{ cm/s}$
$L = 10 \text{ km}$	$U \leq 12 \text{ cm/s}$
$L = 100 \text{ km}$	$U \leq 1.2 \text{ m/s}$
$L = 1000 \text{ km}$	$U \leq 12 \text{ m/s}$
$L = \text{Earth radius} = 6371 \text{ km}$	$U \leq 74 \text{ m/s}$

**TABLE 1-2 LENGTH, VELOCITY, AND TIME SCALES IN THE EARTH'S ATMOSPHERE AND OCEANS**

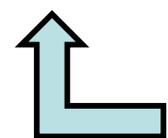
Phenomenon	Length scale $L$	Velocity scale $U$	Time scale $T$
<i>Atmosphere:</i>			
Sea breeze	5–50 km	1–10 m/s	12 h
Mountain waves	10–100 km	1–20 m/s	Days
Weather patterns	100–5000 km	1–50 m/s	Days to weeks
Prevailing winds	Global	5–50 m/s	Seasons to years
Climatic variations	Global	1–50 m/s	Decades and beyond
<i>Ocean:</i>			
Internal waves	1–20 km	0.05–0.5 m/s	Minutes to hours
Coastal upwelling	1–10 km	0.1–1 m/s	Several days
Large eddies, fronts	10–200 km	0.1–1 m/s	Days to weeks
Major currents	50–500 km	0.5–2 m/s	Weeks to seasons
Large-scale gyres	Basin scale	0.01–0.1 m/s	Decades and beyond

} Meso-scale  
} Synoptic scale  
} General Circulation

} Meso-scale



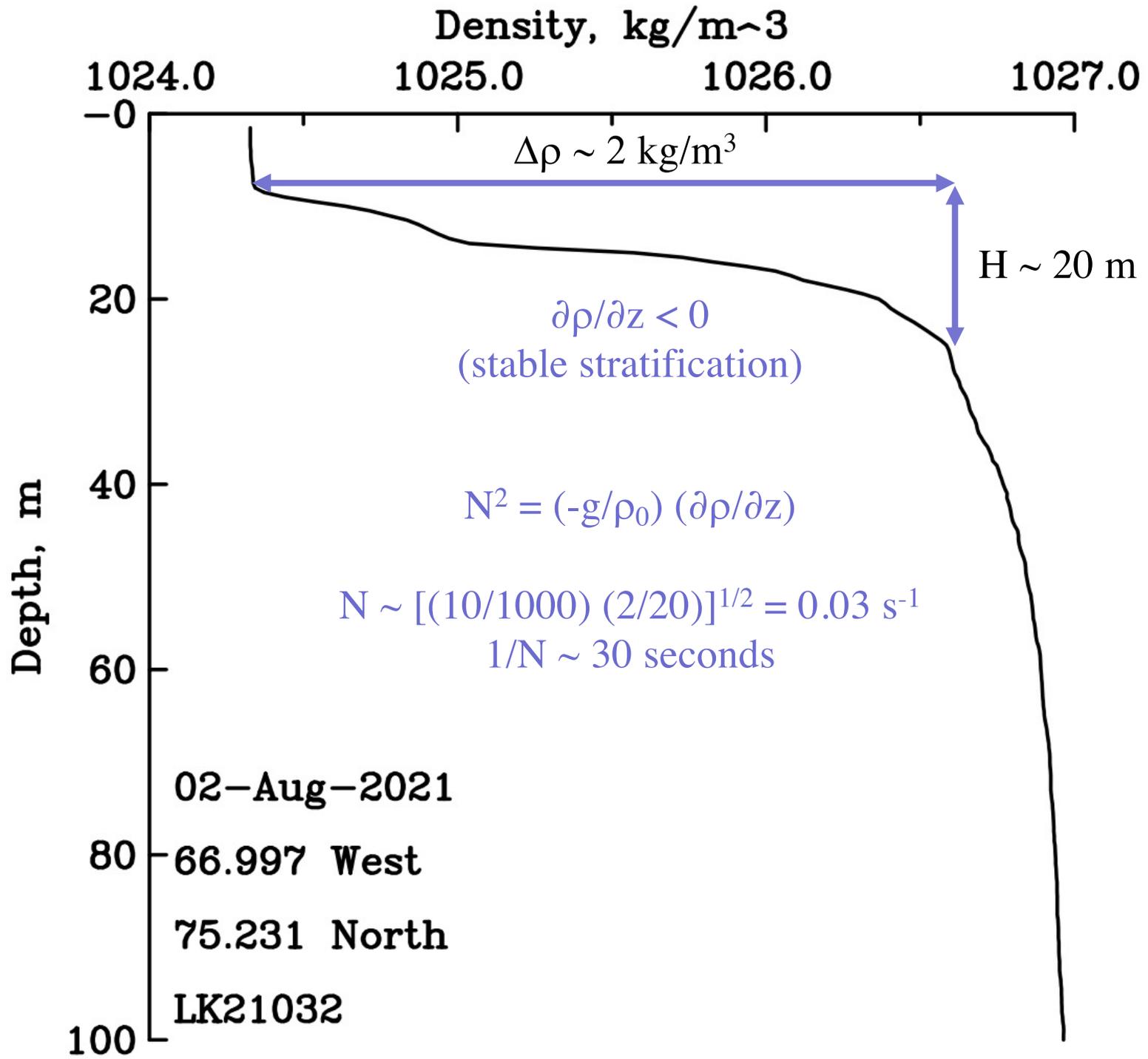
Analogous





Mixing without rotation

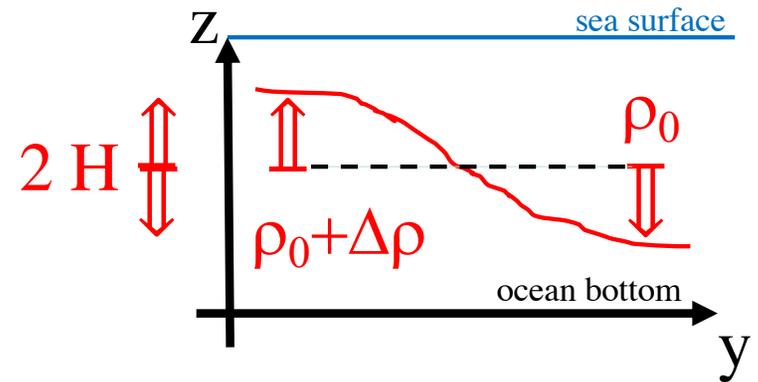
Mixing with rotation



## Role of Density Stratification:

Density = Mass/Volume [kg/m<sup>3</sup>]

Froude Number = kinetic/potential energy



Kinetic Energy is  $\rho_0 U^2 / 2$  per unit volume

Potential Energy

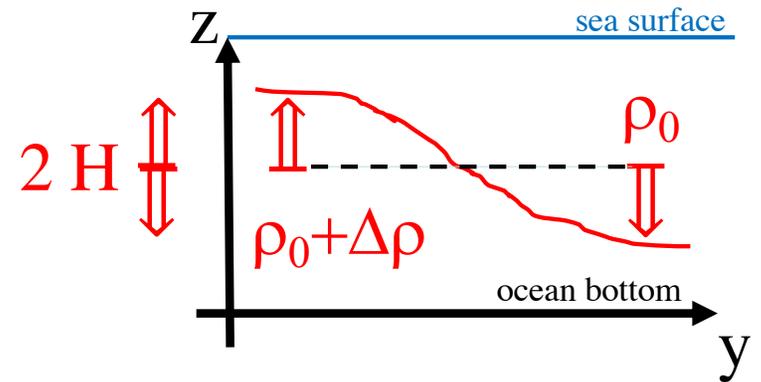
+  $(\rho_0 + \Delta\rho) g H$  of raised parcel with density  $\rho_0 + \Delta\rho$  by scale height  $H$

-  $(\rho_0) g H$  of lowered parcel with density  $\rho_0$  by scale height  $H$

---

=  $\Delta\rho g H$  net gain of Potential Energy (per unit volume)

Froude Number =  $1/2 \rho_0 U^2 / (\Delta\rho g H)$



$$\text{Froude Number} = 1/2 \rho_0 U^2 / (\Delta\rho g H)$$

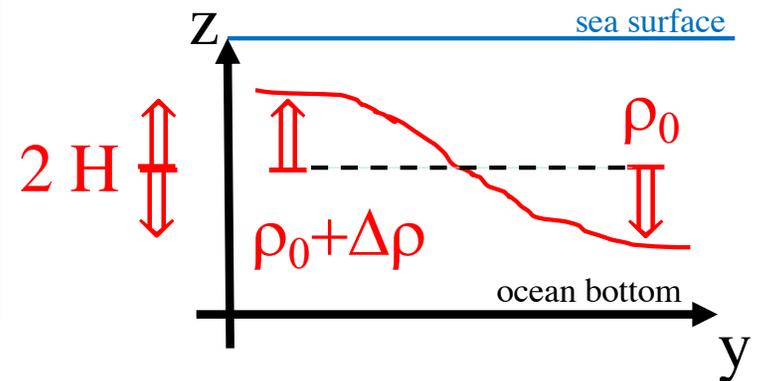
- Fr ~ 1      changing stratification uses substantial kinetic energy
- Fr << 1    insufficient kinetic energy to change stratification  
stratification constrains the flow; stiff density columns
- Fr >> 1    changing stratification requires very little kinetic energy  
stratification not important; hydraulic flows

Froude number can also be interpreted as

$$\text{Fr} = (U/C)^2$$

where U is a particle and  $C = (\Delta\rho/\rho_0 g H)^{1/2}$  is an internal wave phase velocity

Internal Rossby radius of Deformation  
defines  
Length Scale of Synoptic Motions in  
Atmosphere and Oceans



Stratification  $Fr \sim \rho_0 U^2 / (\Delta\rho gH)$  kinetic/potential energy  
Rotation  $Ro \sim (U/L) / \Omega$  rotational/advective time scale

What if stratification and rotation are of similar importance?

$U \sim \Omega L$  from  $Ro \sim 1$

$U^2 \sim \Delta\rho / \rho_0 gH$  from  $Fr \sim 1$

$L \sim U / \Omega$  from  $Ro \sim 1$

$L \sim (\Delta\rho / \rho_0 gH)^{1/2} / \Omega$   $U$  from  $Fr \sim 1$

<i>Earth</i> $g=9.81 \text{ m/s}^2$ $\Omega=7.29 \times 10^{-5} \text{ s}^{-1}$	<i>Atmosphere</i>	<i>Ocean</i>
$\rho_0, \text{ kg/m}^3$	1.2	1028
$\Delta\rho, \text{ kg/m}^3$	0.03	2
H, m	5000	1000
L, km	500	60
U, m/s	30	4

In a given fluid, of mean density  $\rho_0$  and density variation  $\Delta\rho$ , occupying a height  $H$  on a planet rotating at rate  $\Omega$  and exerting a gravitational acceleration  $g$ , the scale  $L$  arises as a preferential length over which motions will take place. On Earth ( $\Omega = 7.29 \times 10^{-5} \text{ s}^{-1}$  and  $g = 9.81 \text{ m/s}^2$ ), typical conditions in the atmosphere ( $\rho_0 = 1.2 \text{ kg/m}^3$ ,  $\Delta\rho = 0.03 \text{ kg/m}^3$ ,  $H = 5000 \text{ m}$ ) and in the ocean ( $\rho_0 = 1028 \text{ kg/m}^3$ ,  $\Delta\rho = 2 \text{ kg/m}^3$ ,  $H = 1000 \text{ m}$ ) yield the following natural length and velocity scales:

$$L_{\text{atmosphere}} \sim 500 \text{ km} \quad U_{\text{atmosphere}} \sim 30 \text{ m/s} \quad (1-7)$$

$$L_{\text{ocean}} \sim 60 \text{ km} \quad U_{\text{ocean}} \sim 4 \text{ m/s.} \quad (1-8)$$

# Physics & Calculus Refresher

mass \* acceleration = Force

Newton's 2<sup>nd</sup> Law

$$m * a = F$$

but F is sum of all forces  
and  $a = D(q)/Dt$

$$m * D(q)/Dt = F_1 + F_2 + \dots + F_n$$

but q and F are vectors  
q = (u, v, w) for velocity  
F = (F<sup>(x)</sup>, F<sup>(y)</sup>, F<sup>(z)</sup>) for forces

$D(q)$  is a differential velocity increment of a particle with velocity q

$$q = q(t, x, y, z) = q(t, x(t), y(t), z(t))$$

as time t changes, so does the location (x,y,z) of the particle

$$D(u) = \partial_t u + \partial_x u * dx + \partial_y u * dy + \partial_z u * dz$$

Chain Rule and  
partial derivatives

$$D(u)/Dt = \partial_t u + \partial_x u * dx/dt + \partial_y u * dy/dt + \partial_z u * dz/dt$$

$$\partial_t u = \partial u / \partial t$$

$$\partial_x u = \partial u / \partial x$$

$$\partial_y u = \partial u / \partial y$$

$$\partial_z u = \partial u / \partial z$$

$$D(u)/Dt = \partial_t u + \partial_x u * u + \partial_y u * v + \partial_z u * w$$