Equations #1, #2, and #3:

Governing Equations

Newton's 2nd law in a rotating frame

$$\frac{d\vec{u}}{dt} + 2\vec{\Omega}\times\vec{u} = \sum_i \vec{F}_i$$

acceleration of fluid particle — sum of all forces applied to that particle

A fluid parcel is defined by

$$\vec{x} = \vec{x}(t, \vec{x}(t=0))$$

and has a velocity

$$\vec{u} = \vec{u}(t, \vec{x}(t=0))$$

or, if we tag it by its trajectory $\vec{x}_p(t)$

$$\vec{u} = \vec{u}(t, x_p(t), y_p(t), z_p(t))$$

then the time-rate-of-change becomes

$$\vec{i}\cdot\frac{d\vec{u}}{dt} = \frac{d}{dt}u = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}$$

$$= \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

velocity of a particle — velocity field through which a particle may move

$$= \vec{i}\cdot\underbrace{\left(\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla})\,u\right)}$$

where $\nabla \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

Lagrangian

Eulerian $\left\{ \frac{D\vec{u}}{Dt} \right.$ advective acceleration (nonlinear)

We have interpret velocity as a velocity field

Thus

$$\frac{D}{Dt}\vec{u} + 2\vec{\Omega}\times\vec{u} = -\frac{1}{\rho}\vec{\nabla}p + \vec{F}_{stress} - \vec{\nabla}\left(\Phi_g \cancel{- |\vec{\Omega}|^2|\vec{x}|^2/2}\right)$$

Φ_c

accelerations — pressure gradient "normal stress" — friction "tangential stress" — $\underbrace{\text{"normal stress" "tangential stress"}}_{\text{appendix - A}}$ — gravity, centrifugal potentials $= \vec{g}$ body force

	$\frac{U}{T}$	$\frac{\Omega U}{}$	$\frac{P}{\rho}$	?	g
	T ~ 1 day				g~9.81 m/s^2
Atm.:	$\frac{10\ m/s}{(24\cdot 3600)s}$	$\frac{10\ m/s}{24\cdot 3600\ \text{seconds}}$			$\frac{10\ m/s}{1\ \text{second}}$
ocean:	$\frac{1\ m/s}{\text{day}}$	$\frac{1\ m/s}{\text{day}}$			$\frac{10\ m/s}{1\ \text{sec}}$

Thus in geophysical flows gravity is usually <u>several</u> orders of magnitudes larger than the accelerations

Hence we define perturbation pressure p' and perturbation density ρ' as departures from an equilibrium (~~one~~ at rest $\Rightarrow$ $\rho' = 0$, $p' = 0$) :

$$p = p_0(z) + p' \quad \text{and} \quad \rho = \rho_0(z) + \rho'$$

that satisfy

$$\boxed{\frac{dp_0}{dz} = -g\rho_0 \qquad \text{in the vertical (hydrostatic)}}$$

(will return to this later)

The perturbation pressure and density then are

Pressure p:
p = p0(z) + p'(x,y,z,t)

Density rho:
rho = rho0(z) + rho'(x,y,z,t)

$$p = p_0(z) \pm p'(x,y,z,t)$$
$$\underbrace{\rho}_{\text{actual}} = \underbrace{\rho_0(z)}_{\text{@ rest}} \pm \underbrace{\rho'(x,y,z,t)}_{\text{perturbation due to motion}}$$

$$\boxed{\rho_0 \gg \rho'}$$

and our momentum balance becomes

not used after Boussinesq

$$\frac{D\vec{u}}{Dt} + 2\vec{\Omega}\times\vec{u} = -\frac{1}{\rho_0}\vec{\nabla}p' - \boxed{\frac{\rho'}{\rho_0}\vec{g}} + \vec{F}_{stess}$$

"new"
not yet explained

This is the Boussinesq Approximation

In the special case of a homogeneous fluid (uniform density)

$$\rho' = 0$$

(barotropic dynamics
no buoyancy forces)

Otherwise $-\rho'\vec{g}$ represents the buoyancy force (baroclinic dynam...

Independent variables : t, x, y, z
Dependent variables : u, v, w, p, ρ

3 equations for 5 dependent variables

$\rightarrow$ need two more independent equations to determine the system's properties