

(26a)

$$\frac{\partial u}{\partial t} + \omega \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} + 2\Omega \cos \phi \cdot w + 2\Omega \sin \phi \cdot u = -\frac{1}{\rho_0} \frac{\partial p}{\partial x}$$

$$+ \sqrt{\frac{\partial^2 u}{\partial x^2}} + \sqrt{\frac{\partial^2 u}{\partial y^2}} + \sqrt{\frac{\partial^2 u}{\partial z^2}}$$

$$\frac{u}{T} \frac{u^2}{L} = \frac{u^2}{L} = \frac{w u}{H} \quad \underbrace{\Omega w \ll \Omega u}_{\text{continuity}} \quad \frac{P}{\rho_0 L} \frac{\gamma u}{L^2} = \frac{\gamma u}{L^2} \leftarrow \frac{\gamma u}{H^2}$$

$\frac{u}{\Omega T} \frac{u^2}{\Omega L} \quad u \quad \frac{P}{\rho_0 L \Omega} \quad \frac{\gamma u}{H^2 \Omega}$

$$\left. \frac{1}{\Omega T} \frac{u}{\Omega L} \quad 1 \quad \frac{P}{\rho_0 L \Omega} \frac{u}{\Omega u} \quad \frac{\gamma}{H^2 \Omega} \quad \frac{P}{\rho_0 L u \Omega} \right) : 2$$

$$\frac{1}{\Omega T} \frac{u}{\Omega L} \quad 1 \quad \frac{P}{\rho_0 L \Omega} \frac{u}{\Omega u} \quad \frac{\gamma}{H^2 \Omega} \quad \frac{P}{\rho_0 L u \Omega}$$

$\frac{P}{\rho_0 u^2 \cdot R_o}$

$P \sim \rho_0 L u \Omega$   
geostrophic scale  
for pressure perturbation

$$R_o \quad R_o \quad 1 \quad ? \quad E_v$$

Rossby #

$$R_o / F_r^2$$

Ekman #

from vertical mom. balance scaling

$$R_o = \frac{u}{\Omega L}$$

$$E_v = \frac{\gamma}{H^2 \Omega}$$

$$F_r = \frac{u}{\sqrt{\frac{\Delta P}{\rho_0 g} H'}}$$

Vertical

perturbation pressure  $p = p_0 + p'$  (265)  
total norma- due to motion

$$\frac{\partial \omega}{\partial t} + u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} + w \frac{\partial \omega}{\partial z} - 2\Omega \cos \phi u = -\frac{1}{\rho_0} \frac{\partial p'}{\partial z} - g \Delta p + \text{friction}$$

(1)      (2)      (3)      (4)

(5)

$\rho_0$

(6)

$\rho_0$

(7)

$v \frac{\partial^2 w}{\partial x^2}$

(8)

$v \frac{\partial^2 w}{\partial z^2}$

(9)

pressure due to motion

no

$$\frac{\partial p_0}{\partial z} = -\rho_0 g$$

excluded  
p. 24

$$\frac{W}{T}, \frac{uW}{L}, \frac{uW}{L}, \frac{W^2}{H}, \frac{\Omega u}{L}, \frac{P}{\rho_0 H}, \frac{g \Delta p}{\rho_0}, \frac{vW}{L^2}, \frac{vW}{H^2}$$

$$\frac{1}{T}, \frac{u}{L}, \frac{u}{L}, \frac{W}{H}, \frac{\Omega u}{W}, \frac{P}{\rho_0 H W}, \frac{g \Delta p}{\rho_0 W}, \frac{v}{L^2} \ll \frac{v}{H^2}$$

$$\frac{1}{T}, \frac{u}{L}, \frac{H}{T \cdot H}, \frac{\Omega u}{H \cdot T}, \frac{P}{\rho_0 H \cdot H/T}, \frac{g \Delta p}{\rho_0 H \cdot H/T}, \frac{v}{L}$$

$W = \frac{1}{T}$

$u = \frac{L}{T}$

$$\frac{1}{T}, \frac{L/T}{L}, \frac{1}{T}, \frac{\Omega L/T}{H/T}, \frac{P}{\rho_0 H^2/T}, \frac{g \Delta p}{\rho_0 H/T}, \frac{v}{L}$$

$$\frac{1}{T}, \frac{1}{T^2}, \frac{1}{T}, \frac{\Omega L}{H}, \frac{P}{\rho_0 H^2/T}, \frac{g \Delta p}{\rho_0 H/T}, \frac{v}{L}$$

$\frac{1}{T} \approx \Omega$

$$(\Omega, \Omega, \Omega) \ll (\Omega, \frac{L}{H}) \frac{P}{\rho_0 H^2 \Omega}, \frac{g \Delta p}{\rho_0 H \Omega}, \frac{v}{L}$$

$\Omega \approx \frac{L}{H}$

$$\frac{\Omega^2 L}{H}$$

$$\frac{P}{\rho_0 H^2}$$

$$\frac{g \Delta p}{\rho_0 H}$$

$$\frac{v \Omega}{L}$$

$H \approx L$   
 $u = -\Omega$

$$\Omega u$$

$$\frac{P}{\rho_0 H}$$

$$\frac{g \Delta p}{\rho_0}$$

$$v \Omega \frac{H}{L}$$

plug in numbers

$$\Omega \sim 1 (\text{day})^{-1} \quad 10^5 \text{ s}^{-1}$$

$$U \sim 1 \text{ m s}^{-1}$$

$$g \sim 10 \text{ m s}^{-2}$$

$$\Delta P \sim 10 \text{ dyne m}^{-3}$$

$$P_0 \sim 10^3 \text{ dyne m}^{-3}$$

$$\gamma \sim 10^{-3} \text{ m}^2 \text{s}^{-1}$$

$$H \sim 10^3 \text{ m}$$

$$L \sim 10^5 \text{ m}$$

So balance between (2) and (3)

*P is scale for perturbation pressure*

$$\frac{P}{P_0 H} \sim g \frac{\Delta P}{P_0} \quad \text{or} \quad P \sim g H \frac{\Delta P}{P_0}$$

*Stop*

$$\text{or } \frac{P}{P_0 U^2} \cdot R_o \text{ becomes } \frac{g H \Delta P}{P_0 U^2} \cdot R_o = \left( \sqrt{\frac{\Delta P}{P_0}} \frac{g H}{U} \right)^2 \cdot R_o$$

$$= \frac{R_o}{F_{\text{Fr}}^2} \cdot R_o / F_{\text{Fr}}^2$$

$$1 / \text{Froude } \#$$

So from the vertical hydrostatic (for perturbation pressure!)

we get

$$\frac{P}{\Delta \rho g H} \sim 1 \quad \text{or} \quad P \sim \Delta \rho g H$$

The scaling of the perturbation pressure in the horizontal

momentum balance was

$$\frac{P}{\rho_0 L U \Omega} \sim \frac{\Delta \rho g H}{\rho_0 L U \Omega} = \underbrace{\frac{\Delta \rho g H}{\rho_0 U^2}}_{\frac{1}{F_r^2}} \cdot \underbrace{\frac{U}{L \Omega}}_{R_o}$$

where

$$F_r = \frac{U}{\sqrt{g \frac{\Delta \rho}{\rho_0} H}} = \frac{\text{flow velocity}}{\text{phase velocity of an internal perturbation}}$$

See also Lecture-2:

Kinematic vs Potential Energy in a stratified fluid

So we got

$$\text{Rossby number } R_o = \frac{U}{L \Omega}$$

$$\text{Ekman number } E_v = \frac{v}{H^2 \Omega}$$

$$\text{Frondle number } Fr = \frac{U}{\sqrt{g \Delta \rho / \rho_0 H}}$$

$$\text{Reynolds number } Re = \frac{UL}{v} = \frac{R_o}{E_v} \left( \frac{L}{H} \right)^2$$

First law of thermodynamics (thermal energy or heat equation)

Kundu (1990)

$$\frac{De}{Dt} = Q - \rho \frac{Dp}{Dt}$$

p.104/105 most useful

internal energy = heat mechanical  
 change received work done  
~~energy gained~~  
 (energy gain)

$$e = C_v T \quad \text{internal energy of a fluid parcel}$$

$$Q = -\frac{1}{\rho} \vec{\nabla} \cdot \vec{q} \quad \begin{aligned} &\text{rate of heat gained, where } \vec{q} \text{ is a heat flux} \\ &\text{and } k \text{ is thermal conductivity} \end{aligned}$$

$$= +\frac{1}{\rho} \vec{\nabla} \cdot (k \vec{\nabla} T) \quad \text{in a Fourier law of heat diffusion}$$

$$\rightarrow \rho C_v \frac{DT}{Dt} = k \nabla^2 T - \rho \nabla \cdot \vec{u} \quad \text{from } \frac{\partial p}{\partial t} + \nabla \cdot (\vec{u} \cdot \rho) = 0$$

under Boussinesq or  $\frac{D}{Dt} p + \rho \nabla \cdot \vec{u} = 0$

approximation

or  $\frac{D(T)}{Dt} = \kappa_T \nabla^2 T \quad \text{heat equation}$

$$\kappa_T = \frac{k}{\rho C_v}$$

is this  $p_0(z)$   
 or  $p(x, y, z, t)$ ?

New equation, but also a new variable "Temperature"

$\rightarrow p_0!$

## Equation #6 for 6 variables u,v,w,p,rho,T

(26)  
(27)  
(28)

### Equation of State

dry air in atmosphere

$$\rho = p / RT$$

ideal gas

$$R = C_p - C_v$$

$\rho_{\text{gas}}$

no such expression exist for the ocean, hence density of seawater is an empirically determined expression, e.g.,

$$\rho = \rho(T, S, p) \approx \rho_0 [1 - \alpha(T - T_0) + \beta(S - S_0)]$$

New equation, but new variable (salinity), so, need an expression for salt (humidity in atmosphere), such as

## Equation #7 for 7 variables

u,v,w,p,rho,T,salt

$$\frac{\partial S}{\partial t} = \kappa_s \nabla^2 S$$

$\kappa_s$  diffusion coefficient  
for salt

→ 7 equations for 7 variables

$$u, v, w, \rho, p, T, S$$

that all depend, generally, on

$$x, y, z, t$$

Need to simplify substantially