Equation #5 for 5 variables u,v,w,p,rho

<u>First law of thermodynamics</u> (thermal energy or heat equation)

Kundu (1990)
p.104/105 most useful

$$\frac{De}{Dt} = Q - \frac{p}{\rho}\frac{D\rho}{Dt}$$

internal energy change ~~energy gained~~ (energy gain) = heat received + mechanical work done

$e = C_v T$ internal energy of a fluid parcel

$Q = -\frac{1}{\rho}\vec{\nabla}\cdot\vec{q}$ rate of heat gained, where $\vec{q}$ is a heat flux and k is thermal conductivity

$= +\frac{1}{\rho}\vec{\nabla}(k\nabla T)$ in a Fourier law of heat diffusion

$$\rightarrow \rho C_v \frac{DT}{Dt} = k\nabla^2 T - p\,\nabla\cdot\vec{u}$$

from $\frac{\partial\rho}{\partial t} + \nabla(\vec{u}\rho) = 0$

under Boussinesq approximation or $\frac{D}{Dt}\rho + \rho\nabla\cdot\vec{u} = 0$

$$\boxed{\text{or } \frac{D(T)}{Dt} = \kappa_T \nabla^2 T \quad \text{heat equation}}$$

$\kappa_T = \frac{k}{\rho C_v}$

is this $\rho_0(z)$ or $\rho'(x,y,z,t)$?
$\rightarrow \rho_0$!

New equation, but also a new variable "Temperature"

Equation #6 for 6 variables u,v,w,p,rho,T

Equation of State

dry air in atmosphere $\rho = p / RT$ ideal gas

$R = C_p - C_v$

no such expression exist for the ocean, hence density of seawater is an empirically determined expression, e.g.,

$$\rho = \rho(T, S, p) \approx \rho_0 \left[1 - \alpha (T - T_0) + \beta (S - S_0) \right]$$

New equation, but new variable (salinity), so, need an expression for salt (humidity in atmosphere), such as

Equation #7 for 7 variables
u,v,w,p,rho,T,salt

$$\frac{D}{Dt} S = \kappa_s \nabla^2 S$$

κ_s diffusion coefficient for salt

$\rightarrow$ 7 equations for 7 variables

u, v, w, ρ, p, T, S

that all depend, generally, on

x, y, z, t

Need to simplify substantially