$\boxed{G7.3}$

Gill (1982) chapt. 7.3

The complete problem is solved if we find complete solutions to

c^2 = g*H

$$\frac{\partial^2 \eta}{\partial t^2} \underbrace{- c^2\left(\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2}\right)}_{\text{relative vorticity}} + \underbrace{f^2 \eta}_{\text{stretching}} = -f^2 \eta_0 \operatorname{sgn}(x)$$

Consider this complete solution to consist of two parts, that is

	Waves		geostrophic flow
$\eta(x,y,t) =$	$\eta_H(x,y,t)$	+	η_{steady}
general solution of the forced (inhomogeneous) problem	general solution of the unforced (homogeneous) problem		particular solution of the forced problem

Which gives

We got this "steady" solution last class period when we used initial conditions at time t=0:

(a) forced problem $\quad - c^2\left(\frac{\partial^2 \eta_{steady}}{\partial x^2} + \frac{\partial^2 \eta_{steady}}{\partial y^2}\right) + f^2 \eta_{steady} = -f^2 \eta_0 \operatorname{sgn}(x)$

(b) unforced problem $\quad \frac{\partial^2}{\partial t^2}\eta_H - c^2\left(\frac{\partial \eta_H}{\partial x^2} + \frac{\partial \eta_H}{\partial y^2}\right) + f^2 \eta_H = 0$

We solved (a) that gave us $\boxed{\eta_{steady} = \eta_0 \operatorname{sgn}(x)\left(-1 + e^{-|x|/a}\right)}$

We still need to solve (b) that give us the time dependant or transient part of the complete solution

Recall that "a" in the exponential is
a = sqrt(g*H)/f

For initial condition we also have

$$\eta(x,y,t=0) = \eta_H(x,y,t=0) + \eta_{steady}(x,y)$$

or $$\eta_H(x,y,t=0) = -\eta_0 \, \mathrm{sgn}(x) - \eta_0 \, \mathrm{sgn}(x)\left(-1 + e^{-|x|/a}\right)$$

$$\eta_H(x,y,t=0) = -\eta_0 \, \mathrm{sgn}(x) \, e^{-|x|/a}$$

The time-dependent transients are obtained subject to this condition from

$$\boxed{\frac{\partial^2 \eta_H}{\partial t^2} - c^2\left(\frac{\partial^2 \eta_H}{\partial x^2} + \frac{\partial^2 \eta_H}{\partial y^2}\right) + f^2 \eta_H = 0}$$

linear and unforced wave equation with a **wrinkle**

and solutions are of the form

Wave Solutions:

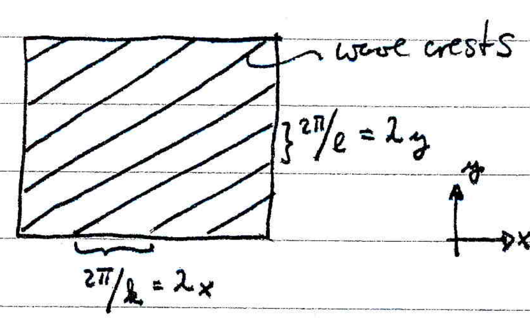
$$\eta_H \propto \exp\left[i\left(kx + ly - \omega t\right)\right]$$

complex i=sqrt(-1) exponentia

exp[i*wt]=cos(w*t)+i*sin(w*t)

where $\vec{k} = (k, l)$ is the horizontal vector wave number

with $\kappa_H^2 = k^2 + l^2 = |\vec{k}|$



G8.2

Gill (1982)
chapt. 8.2

Poincare solutions

too much technical detail

removed 2024

Dispersion Relation

$$\boxed{\omega^2 = f^2 + \kappa_H^2 c^2}$$

(w/f)^2 = 1 + kappa^2 * (c/f)^2
(w/f)^2 = 1 + kappa^2* a^2

Recall
c^2 = g*H
c/f = a

short Poincaré waves ($\kappa_H a \gg 1$)

Rossby radius a >> wavelenth 1/kappa

have

$$\omega \approx \kappa_H c = k c$$

thus ellipticity $\frac{\omega}{f} \approx \frac{k c}{f} = k \cdot a \gg 1$ long + thin almost rectilinear

long Poincaré waves ($\kappa_H a \ll 1$)

have $\omega \approx f$

thus ellipticity is $\frac{\omega}{f} \approx \frac{f}{f} = 1$ circular motion

sense of rotation always clockwise (anti-cyclonic)

show Gill Fig. 8.2 and 8.3

then move to 9.3 : Forcing due to surface stress