

$u = 0$ (no across-shore velocity):

$$\frac{\partial \zeta}{\partial t} - f v = -g \frac{\partial \eta}{\partial x} \quad \begin{matrix} \text{geographic} \\ \text{across-shore} \end{matrix}$$

$$\frac{\partial \omega}{\partial t} + f u = -g \frac{\partial \eta}{\partial y}$$

$$-f v + g \frac{\partial \eta}{\partial x} = 0 \quad x-\text{mom}$$

$$\frac{\partial \omega}{\partial t} + g \frac{\partial \eta}{\partial y} = 0 \quad y-\text{mom}$$

$$\frac{\partial \eta}{\partial t} + H \frac{\partial \eta}{\partial x} + H \frac{\partial \omega}{\partial y} = 0$$

$$\boxed{\frac{\partial \eta}{\partial t} = -H \frac{\partial \omega}{\partial y}}$$

relative vorticity

$$\frac{\partial \eta_{\text{mean}}}{\partial x} - \frac{\partial \omega_{\text{mean}}}{\partial y},$$

$$\xi = \frac{\partial \omega}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial \omega}{\partial x}$$

$$\boxed{\frac{\partial}{\partial x} \frac{\partial \omega}{\partial t}} + g \frac{\partial^2 \eta}{\partial x \partial y} + \boxed{\frac{f \partial \omega}{\partial y}} - g \frac{\partial^2 \eta}{\partial x \partial y} = 0$$

$$\boxed{\frac{\partial}{\partial t} \left(\frac{\partial \omega}{\partial x} \right)} + \boxed{f \cdot (-) \frac{\partial \eta}{H \partial t}} = 0$$

from continuity

$$\boxed{\frac{\partial}{\partial t} (\xi)} + f \frac{\partial \eta}{H \partial t} = 0$$

$$\boxed{\frac{\partial}{\partial t} \left(\xi - \frac{f}{H} \eta \right)} = 0$$

potential vorticity
conservation

$$\frac{\partial}{\partial x} x\text{-mom} + \frac{\partial}{\partial y} y\text{-mom}$$

$$-f \frac{\partial v}{\partial x} + g \frac{\partial^2 \eta}{\partial x^2} + \frac{\partial v}{\partial t \partial y} + g \frac{\partial^2 \eta}{\partial y^2} = 0$$

$$-f \frac{\partial v}{\partial x} + g \left(\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} \right) + \frac{\partial}{\partial t} \left(-\frac{1}{H} \frac{\partial \eta}{\partial t} \right) = 0$$

$$\boxed{\frac{\partial^2 \eta}{\partial t^2} - g H \left(\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} \right) + f H \xi = 0}$$

Wave Equation

Two eq. for 2 unknowns

same two equations as before for the geostrophic adjustment.

New Way

to solve: $\frac{\partial}{\partial t} (y\text{-mom}) \rightarrow \frac{\partial^2 v}{\partial t^2} + g \frac{\partial^2 \eta}{\partial y \partial t} = 0$

$$g \cdot \frac{\partial}{\partial y} (\text{continuity}) \rightarrow g \frac{\partial^2 \eta}{\partial t \partial y} + g H \frac{\partial^2 v}{\partial y^2} = 0$$

$$\frac{\partial}{\partial t} (y\text{-mom}) - g \frac{\partial}{\partial y} (\text{continuity}) :$$

$$\boxed{\frac{\partial^2 v}{\partial t^2} - g H \frac{\partial^2 v}{\partial y^2} = 0}$$

1-D wave equation with phase speed $c = \sqrt{g H}$

Solution are $v = V_1(x, y+ct) + V_2(x, y-ct)$

$\underbrace{}$ wave propagating $\underbrace{}$ wave propagating

-y direction +y direction

$$v = V_1(x, y+ct) + V_2(x, y-ct)$$

From

y-momentum: $\frac{\partial v}{\partial t} = -g \frac{\partial \eta}{\partial y}$ or $\frac{\partial v}{\partial y} = -\frac{1}{H} \frac{\partial \eta}{\partial t}$ continuity

$$\eta = -\frac{\sqrt{gH}}{g} V_1 + \frac{\sqrt{gH}}{g} V_2$$

verify by
differentiate with chain rule
 $V(x, y) = y - c^*t$
 $dV/dt = dV/dy * dy/dt$

and geostrophy

x-momentum: $-fv = -g \frac{\partial \eta}{\partial x}$

gives

$$fV_1 + fV_2 = -\sqrt{gH} \frac{\partial V_1}{\partial x} + \sqrt{gH} \frac{\partial V_2}{\partial x}$$

or $fV_1 = -\sqrt{gH} \frac{\partial V_1}{\partial x}$ and $fV_2 = +\sqrt{gH} \frac{\partial V_2}{\partial x}$

Simple ordinary
differential equation
with exponential solutions:

$$V_1 = V_{10}(y+ct) e^{-x/a}$$

$$V_2 = V_{20}(y-ct) e^{+x/a}$$

$$a = \frac{\sqrt{gH}}{f}$$

$$v = V_{10} (g + ct) e^{-x/a}$$

that was the solution propagating
into the negative y -direction

$$\eta = -\frac{\sqrt{gH}}{g} V_{10}(g+ct) e^{-x/a}$$

thus

$$\frac{\eta}{v} = \frac{-\sqrt{gH}}{g} H = -\frac{\sqrt{gH}}{gH} \cdot H = -\frac{H}{\sqrt{gH}}$$

or

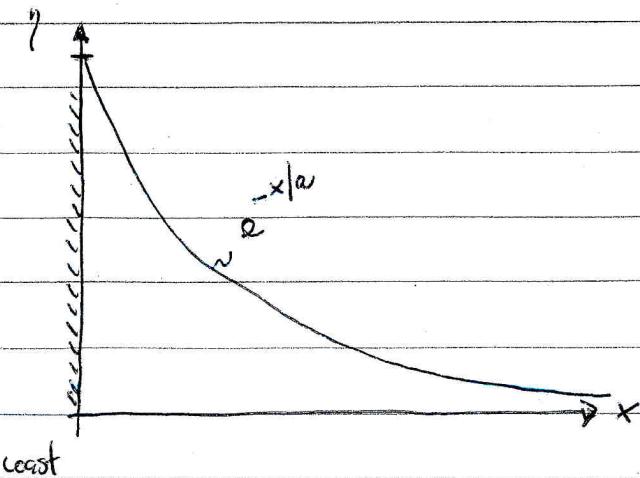
$$\frac{v}{\eta} = -\frac{H}{\sqrt{gH}}$$

and

$$v(x, y, t) = \sqrt{gH} \cdot F(y+ct) \cdot e^{-x/a}$$

$$\eta(x, y, t) = -H \cdot \underbrace{F(y+ct)}_{\text{non-dimensional}} \cdot e^{-x/a}$$

can be any function [a $\cos(y+ct)$ for example]



coastally trapped wave

kinetic energy
focused near
a spatial
feature

other "trapping"

surface trapped

bottom trapped

pycnocline trapped

frontally trapped

| + shows two shades (English Channel front + Dispersion) |