Rossby Wave Eq.
(same as before)

$$\boxed{\frac{\partial \eta}{\partial t} - a^2 \frac{\partial}{\partial t}\left(\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2}\right) - \beta_0 a^2 \frac{\partial \eta}{\partial x} = 0}$$

solutions of the form $\eta \propto \exp[i(lx + my - \omega t)]$
gives

$$\cancel{-i}\omega - a^2(-\cancel{i}\omega)\left[(il)^2 + (im)^2\right] - \beta_0 \frac{a^2\, il}{-i} = 0$$

$$\omega\left[1 - a^2(-)(l^2+m^2)\right] + \beta_0 a^2 l = 0$$

Rossby
Wave
Dispersion

so $$\boxed{\omega = \frac{-\beta_0 a^2 l}{\boxed{1 + a^2(l^2+m^2)}}} = \frac{-\beta_0 l}{a^{-2} + (l^2+m^2)} \overset{m=0}{=} \frac{-\beta_0 l}{a^{-2}+l^2}$$

$$c_x = \frac{\omega}{l} = -\frac{\beta_0 a^2}{\boxed{1 + a^2(l^2+m^2)}}$$

always negative
that is westward phase
propagation ALWAYS

fastest possible phase speed for $\boxed{(a\kappa_H)^2 \ll 1 \quad \text{or} \quad a^2 \ll \kappa_H^{-2} \quad \text{long waves}}$
is

$$c_{max} = -\beta_0 a^2 = -\frac{\beta_0}{f_0^2}\, gH \approx 10 \text{ m/s}$$ in 1000 m of water

$$\beta_0 = \frac{2\Omega\cos\phi}{R} \approx \frac{7.292\cdot 2\cdot\cos\phi}{6300\cdot 10^3 \text{ m s}} \approx \frac{f_0}{6\times 10^6} \approx 10^{-11} \text{ m}^{-1}\text{s}^{-1}$$

$$\frac{\beta_0}{f_0^2} \approx \frac{10^{-11} \text{ m}^{-1}\text{s}^{-1}}{10^{-8} \text{ s}^{-2}} \approx 10^{-3} \text{ m}^{-1}\text{s}$$

$c = \sqrt{gH} \approx 100$ m/s

$$\text{so } \boxed{\frac{\beta_0}{f_0^2}\cdot c^2} \approx 10^{-3} \text{ m}^{-1}\text{s}\cdot 10^4 \frac{\text{m}^2}{\text{s}^2} \approx 10 \frac{\text{m}}{\text{s}}$$

$$\omega = - \frac{\beta_o R^2 k}{\boxed{1 + R^2 \kappa_H^2}}$$

R = a = sqrt(gH)/f Rossby Radius

$\boxed{(a\kappa)^2 \gg 1 \quad \text{or} \quad a \gg \kappa_H^{-1} \quad \text{short waves}}$

$$\boxed{\omega \approx - \frac{\beta_o}{\kappa_H^2} \cdot k}$$

$$\omega \approx - \beta_o \left(\frac{k}{k^2 + l^2} \right)$$

$\boxed{(a\kappa)^2 \ll 1 \quad \text{or} \quad a \ll \kappa_H^{-1} \quad \text{long waves}}$

$$\boxed{\omega \approx - \beta_o R^2 k}$$

$$c_p = \frac{\omega}{k} = - \beta_o R^2 \quad \text{non-dispersive}$$

$$c_x = \frac{\omega}{k} = - \beta_o R^2$$

$$c_g = \frac{\partial \omega}{\partial k} = - \beta_o R^2 = c_p$$

Discuss Dispersion Relations

1. All Rossby waves propagate their phase to the West;
2. Mexican Hat for omega=omega(k,l)
3. omega=omega(k,l=0)
4. For each frequency omega, there exists both a short and a long wave;
5. Long waves are non-dispersive, i.e., phase velocity is the same for all such waves;
6. Short waves are dispersive, i.e., phase velocity varies with wave numbers;
7. A maximal frequency exists at which the eastward component of the group velocity is zero;
8. Long waves propagate their energy to the West (negative group velocity);
9. Short waves propagate their energy to the East (positive group velocity);
10. Group and phase velocities can always have components to theNorth or South.