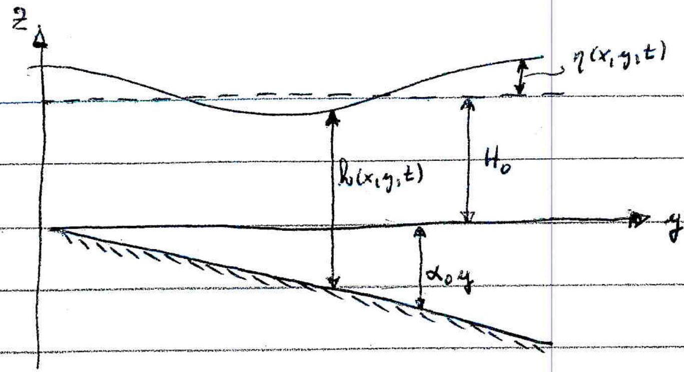
Bottom Slope has the same effect on Rossby Waves as does f=f(latitude)

——> Topographic Rossby Waves (shallow ~ "high latitude")



$$h = H_0 + \alpha_0 y + \eta(x,y,t)$$

with

$$\alpha = \frac{\alpha_0 L}{H} \ll 1$$

Continuity equation writes (depth-integrated)

$$\frac{\partial h}{\partial t} + \frac{\partial(uh)}{\partial x} + \frac{\partial(vh)}{\partial y} = 0$$

or

$$\frac{\partial \eta}{\partial t} + \left(u\frac{\partial \eta}{\partial x} + v\frac{\partial \eta}{\partial y}\right) + H_0\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \alpha_0 y\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \eta\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \alpha_0 v = 0$$

$\frac{L}{T} = L\alpha f$

$$O\left(\frac{\Delta H}{H}\right) \quad O\left(R_o \frac{\Delta H}{H}\right) \quad O(R_o) \quad O(\alpha R_o) \quad O\left(R_o \frac{\Delta H}{H}\right) \quad O(\alpha)$$

Hence for $R_o \ll \frac{\Delta H}{H} \sim R_{o_T} \sim \alpha \ll 1$ ← Think of these conditions as "filters", e.g.,

$$R_{o_T} = \frac{1}{\Omega T}$$

& $\alpha \sim R_{o_T}$ means that

$$\omega \sim \frac{1}{T} \sim \alpha \Omega \sim \alpha f \ll f$$

subinertial motions

we get

$$\frac{\partial \eta}{\partial t} + H_0\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \alpha_0 v = 0$$

Momentum equations and arguments the same as for the Rossby wave development, e.g.,

introduce $u = u_0 + \alpha u_1 + O(\alpha^2)$ etc.

&

$O(1)$ momentum is geostrophic → $O(\alpha)$ momentum is <u>not</u> geostrophic

$O(1)$ continuity is non-divergent → $O(\alpha)$ continuity is <u>not</u> divergent-free

These are the "linear, quasi-geostrophic momentum equations" Gill (1982) p. 497

$O(\alpha)$ x-momentum gives $v_1 = \ldots \quad \Big| \dfrac{\partial}{\partial y}\cdot$

y-momentum gives $u_1 = \ldots \quad \Big| \dfrac{\partial}{\partial x}\cdot$

same as for Rossby wave where the right hand side contains terms involving geostrophic velocity u_0, v_0 only

place these into the continuity to get

$$\boxed{\underbrace{\frac{\partial \eta_1}{\partial t} - a^2 \frac{\partial}{\partial t} \nabla^2 \eta_0 - \beta_0 \boxed{a^2 \frac{\partial \eta_0}{\partial x}}}_{\text{Rossby Wave}} + \alpha_0 \underbrace{\boxed{a^2 \frac{\partial \eta_0}{\partial x}}}_{\text{*new term due to sloping bottom}} \cdot \frac{f}{H_0} = 0}$$

for the special case that deep water is North and shallow water is South

For $f = \text{const.}$ or $\beta \ll \alpha \ll 1$

$$\boxed{\frac{\partial \eta_1}{\partial t} - a^2 \frac{\partial}{\partial t} \nabla^2 \eta_0 + \alpha_0 a^2 \cdot \frac{f}{H_0} \frac{\partial \eta_0}{\partial x} = 0}$$

Topographic Rossby Wave

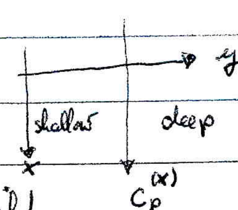
$\eta \propto \exp[i(kx + ly - \omega t)]$

Dispersion relation is

$$\boxed{\omega = \frac{\alpha_0 g}{f} \frac{k}{1 + a^2(k^2 + l^2)}}$$

is identical to that of the Rossby wave as α_0 carries a sign (as does f) indicating that the phase velocity in the x-direction

$$C_p^{(x)} = \frac{\omega}{k} = +\frac{\alpha g}{f} \cdot \frac{1}{1 + a^2 k_H^2}$$



is positive, that is for $f > 0$ with shallow water on the right

Recall the conservation of potential vorticity

$$\frac{D}{Dt}\left(\frac{f+\xi}{h}\right) = 0 \qquad q = \frac{f+\xi}{h}$$

that was derived for a more general case $\quad R_o \sim R_{o_T} \sim O(1)$

$E_V \ll 1 \qquad \rho' = 0$

without any reference to the particulars of
f or h

Here with $\quad f = f_o + \beta_o y \qquad \beta_o L / f_o \ll 1$

and $\quad h = H_o + \alpha_{01} x + \alpha_{02} y \qquad \frac{\alpha_o L}{H_o} \ll 1$

The potential vorticity becomes

$$\frac{D}{Dt}\, q = \frac{f_o + \beta_o y + \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}}{H_o + \alpha_{01} x + \alpha_{02} y + \eta} = \text{const. following a fluid column}$$

provide for pot. vorticity gradients!

$$\cong \underbrace{f_o + \beta_o y}_{\text{ambient planetary vorticity}} \underbrace{- \frac{\alpha_{01} f_o}{H_o} x - \frac{\alpha_{02} f_o}{H_o} y}_{\text{vortex tube stretching by bottom}} + \underbrace{\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}}_{\text{rel. vorticity}} \underbrace{- \frac{f_o}{H_o}\eta}_{\text{vortex tube stretching by free surface}}$$

- It has $\omega_{max} = \frac{\alpha_o g}{2 f a}$
- Long waves have high frequency (almost non-dispersive
- Short waves have low frequency (highly dispersive)