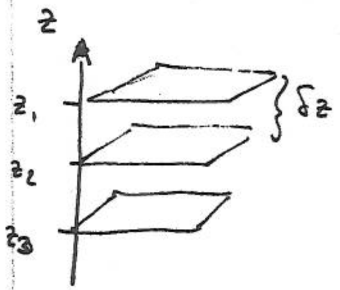
Best to first have the actual vertical velocity distribution as well. start with my data problem.

C.8
GFD

G9.2

Horizontal stress $(\tau^{(x)}, \tau^{(y)})$ at earth's surface

force / unit area



stress applied at one level sets fluid in motion thus exerting a stress on the layer δz below

the stress on the layer below is

$$\left(\tau^{(x)} - \delta z \frac{\partial \tau^{(x)}}{\partial z} \;,\; \tau^{y} - \delta z \frac{\partial \tau^{(y)}}{\partial z} \right)$$

but the lower layer exerts a stress $(-\tau^{(x)}, -\tau^{(y)})$ on the upper layer, thus, the net force/unit area will be the difference, that is,

$$\left(\frac{\partial \tau^{(x)}}{\partial z} , \frac{\partial \tau^{(y)}}{\partial z} \right) . \delta z \qquad \Big| \cdot \frac{\delta x \cdot \delta y}{\text{mass}}$$

$$\downarrow \qquad \underset{\text{density}}{\frac{1}{\rho}} \left(\frac{\partial \tau^{(x)}}{\partial z} , \frac{\partial \tau^{(y)}}{\partial z} \right)$$

This is the additional force / mass that tends to accelerate the fluid

$$E_V = \frac{A_V/\Omega}{H^2} \equiv \left(\frac{\delta_E}{H}\right)^2$$

Linearized momentum equations $Ro \ll 1$, $Ro_T \sim E_V \sim 1$

$u = u(x,y,z,t)$
$v = v(x,y,z,t)$
$\rho = \text{const.}$

$$\frac{\partial u}{\partial t} - f v = \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial \tau^{(x)}}{\partial z}$$

$$\underbrace{\frac{\partial v}{\partial t} + f u}_{\text{Accelerations}} = \underbrace{\underbrace{\frac{1}{\rho}\frac{\partial p}{\partial y}}_{\substack{\text{horizontal}\\ \text{pressure}\\ \text{gradient}}} + \underbrace{\frac{1}{\rho}\frac{\partial \tau^{(y)}}{\partial z}}_{\substack{\text{vertical}\\ \text{stress}\\ \text{gradients}}}}_{\text{Forcing}}$$

Separate into components

[linear equations; solutions can always be added up in the end even though they are considered separately]

$$u = \boxed{u_p} + \boxed{u_E} \quad , \quad v = \boxed{v_p} + \boxed{v_E}$$

$$\boxed{\frac{\partial u_p}{\partial t} + f v_p = -\frac{1}{\rho}\frac{\partial p}{\partial x} \quad , \quad \frac{\partial v_p}{\partial t} - f u_p = -\frac{1}{\rho}\frac{\partial p}{\partial y}}$$

Ekman velocities

$$\boxed{\frac{\partial u_E}{\partial t} + f v_E = +\frac{1}{\rho}\frac{\partial \tau^{(x)}}{\partial z} \quad , \quad \frac{\partial v_E}{\partial t} - f u_E = +\frac{1}{\rho}\frac{\partial \tau^{(y)}}{\partial z}}$$

The stress $(\tau^{(x)}, \tau^{(y)})$ is zero outside the boundary layer, so vertical integration gives

$$\rho\left(\frac{\partial U_E}{\partial t} - f V_E\right) = -\tau_0^{(x)} \quad ; \quad \rho\left(\frac{\partial V_E}{\partial t} + f U_E\right) = -\tau_0^{(y)}$$

sign determines if bottom or surface boundary

"+" boundary is above

"−" boundary is below

and

$$\boxed{(U_E, V_E) = \int (u_E, v_E)\, dz} = \int (u - u_p, v - v_p)\, dz$$

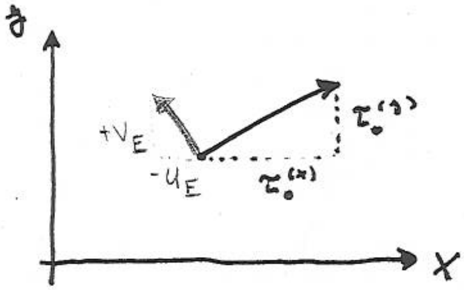
is the volume transport, relative to the pressure gradient flow =
(Ekman volume transport).

For steady state conditions $\frac{\partial}{\partial t} \cdot = 0$ we have

y-component of stress
~
x-component of velocity

$$-\rho f V_E = -\tau_0^{(x)} \quad \text{and} \quad \rho f U_E = -\tau_0^{(y)}$$

x-component of stress
~
y-component of velocity



or $U_E \propto -\tau^{(y)}$
$V_E \propto +\tau^{(x)}$

or $\tau^{(x)} \propto V_E$
$\tau^{(y)} \propto -U_E$

Bottom Boundary Layers to the left of stress (northern hemisphere, $f > 0$)