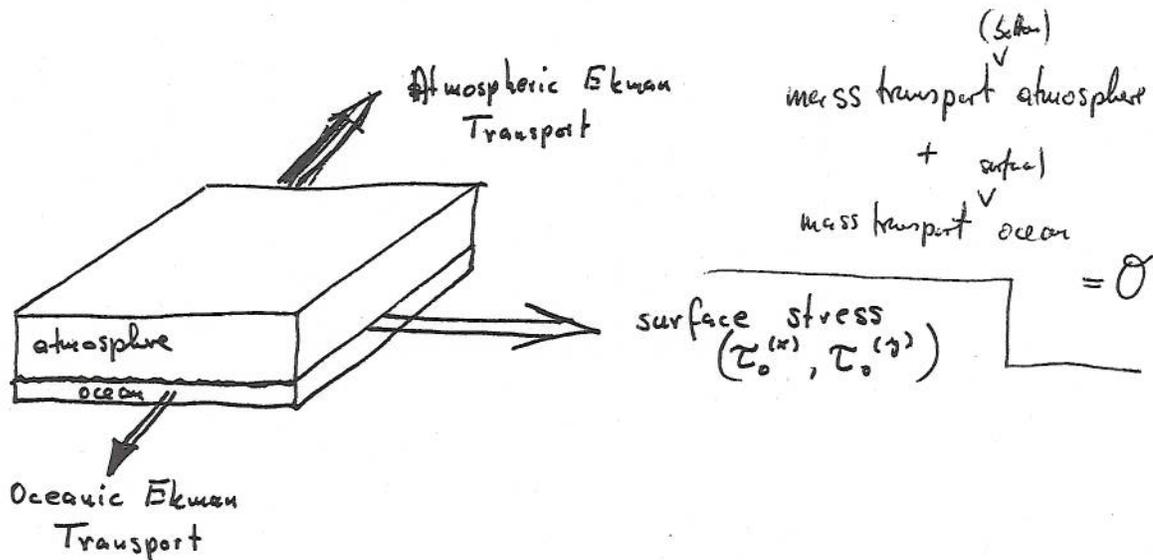


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GFD



Recall that the right/left angle rule applies only to steady conditions!

G.93

Example: Ocean at rest, sudden wind stress $\tau_0^{(x)} = \tau_0^{(y)} = 0$
How does the Ekman transport vary?

$$\frac{\partial (U_E + iV_E)}{\partial t} + if(U_E + iV_E) = \frac{1}{\rho} \tau_0$$

(x-mom) + i(y-momentum) $W = U_E + iV_E$

$$\dot{W} + ifW = \frac{1}{\rho} \tau_0^{(x)}$$

has solution

$$W(t) = -i \left(\frac{\tau_0}{\rho f} \right) \left(1 - e^{-ift} \right)$$

$$= -i \left(\frac{\tau_0}{\rho f} \right) \left(1 - (\cos(ft) - i \sin(ft)) \right)$$

Real part:

$$u_E = \frac{\tau_0}{\rho f} \sin ft$$

Imaginary part:

$$v_E = \frac{\tau_0}{\rho f} (\cos ft - 1)$$

for small (ft)
to $O(ft)$

$$\sin ft \approx ft$$

$$u_E \propto ft$$

and $\cos ft \approx 1$

$$v_E \approx 0$$

thus for $t \ll f^{-1}$ flow is in the direction of stress

small time ft later:

to $O((ft)^2)$

$$\sin ft \approx ft - \frac{(ft)^3}{6}$$

$$\text{and } \cos ft \approx 1 - \frac{(ft)^2}{2}$$

$$u_E \propto ft$$

$$v_E \propto -(ft)^2$$

The flow begins to veer to the right as $v_E < 0$

Note also that v_E contains both

a time-invariant part and

a time-varying part

The steady state part is the EKMAN TRANSPORT

The time-varying part is an INERTIAL OSCILLATION

Shaw Gill (1982) Fig. 8.3 and Fig. 9.2

The solution

$$W(t) = -i \frac{\tau_0}{\rho f} \left(1 - e^{-ift} \right)$$

satisfies the initial condition $W(t=0) = 0$

→ and consists of a steady state part = $-i \tau_0 / \rho f$ (imaginary!) thus \perp to winds

→ and an oscillatory component = $+i \frac{\tau_0}{\rho f} e^{-ift}$

that can be interpreted as a very long Poincaré wave (inertial oscillation)

If $W(t=t_0) = W_0$ and the wind stress τ_0 changes at that time, we get the solution

$$W(t) = \underbrace{-i \frac{\tau_0}{\rho f} \left(1 - e^{-if(t-t_0)} \right)}_{\text{same as before}} + \underbrace{W_0 e^{-if(t-t_0)}}_{\text{correction to accommodate the new initial condition}}$$

or

$$W(t) = \underbrace{-i \frac{\tau_0}{\rho f}}_{\text{steady state "Ekman layer" response}} + \underbrace{\left(i \frac{\tau_0}{\rho f} + W_0 \right)}_{\text{amplitude of inertial oscillations}} e^{-if(t-t_0)}$$

steady state
"Ekman layer"
response

amplitude
of inertial
oscillations

if $W_0 = -i \frac{\tau_0}{\rho f}$ then there are no i.o.