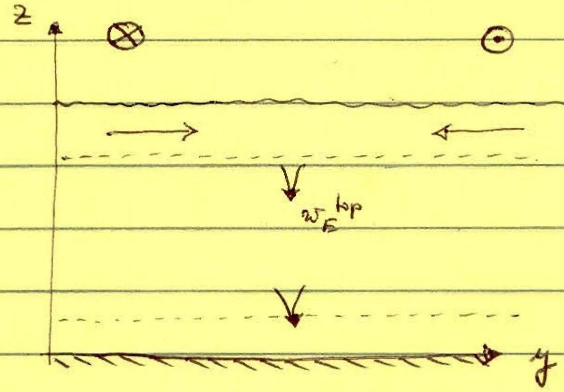
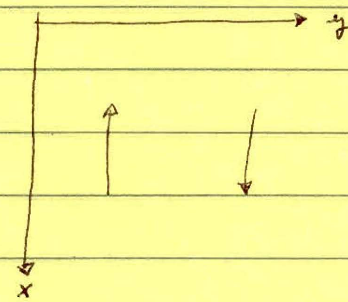
Large Scale Ocean Circulation (steady state)





$$w_E^{top} = \frac{\partial}{\partial x}\left(\frac{\tau^{(y)}}{\rho f}\right) - \frac{\partial}{\partial y}\left(\frac{\tau^{(x)}}{\rho f}\right)$$

Ekman pumping due to wind-stress curl

$$w_E^{bottom} = \left(\frac{f}{|f|}\right)\frac{d}{2}\left(\frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y}\right)$$

Ekman pumping due to bottom friction

For the geostrophic interior, we have $[u_g = u\ , v_g = v]$

notice that there is NO vertical dependence !

$$\left\{\begin{aligned} -(f_0+\beta_0 y)\,v &= -\frac{1}{\rho}\frac{\partial p}{\partial x} \quad \Big|\ -\frac{\partial}{\partial y} \\ +(f_0+\beta_0 y)\,u &= -\frac{1}{\rho}\frac{\partial p}{\partial y} \quad \Big|\ +\frac{\partial}{\partial x} \\ 0 &= -\frac{1}{\rho_0}\frac{\partial p}{\partial z} \end{aligned}\right.$$

$\beta = \beta_0 L/f_0 \ll 1$

$\therefore \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ does not depend on z either nor does, by inference $\frac{\partial w}{\partial z}$!

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

taking the curl of momentum $\longrightarrow$

$$f_0\Big(1 + \underbrace{\frac{\beta_0 y}{f_0}}_{\ll 1}\Big)\underbrace{\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)}_{-\frac{\partial w}{\partial z}} + \beta_0 v = 0 \quad \Downarrow \quad \boxed{f_0\frac{\partial w}{\partial z} = \beta_0 v}$$

linearized vorticity equation

No vertical dependence in (x, y, z) momentum

$\to$ (u, v, w) do not depend on z!

$\to$ $$\frac{\partial w}{\partial z} = \frac{w_E^{top} - w_E^{bottom}}{H}$$

$$\frac{\partial w}{\partial z} = \frac{1}{\rho f_0 H} \vec{k} \cdot \nabla \times \vec{\tau} - \frac{d}{2H} \frac{f_0}{|f_0|} \xi_g$$

geostrophic rel. vorticity:

$$\xi_g = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{1}{\rho f_0} \nabla^2 p$$

2nd derivative of pressure

into the linearized vorticity equation which is

geostrophic velocity $v = \frac{1}{\rho f_0} \frac{\partial p}{\partial x}$

$$\boxed{\frac{\partial w}{\partial z}} = \boxed{\frac{\beta_0 v}{f_0}} = \underbrace{\frac{\beta_0}{f_0} \frac{1}{\rho f_0} \frac{\partial p}{\partial x}}$$

and we get

$$\boxed{\underbrace{\frac{1}{\rho f_0 H} \left(\vec{k} \cdot \nabla \times \vec{\tau} \right)}_{\substack{\text{wind-stress curl} \\ \text{(surface Ekman)}}} - \underbrace{\frac{d}{2H} \frac{\cancel{f_0}}{|f_0|} \frac{1}{\rho \cancel{f_0}} \nabla^2 p}_{\substack{\text{relative vorticity} \\ \text{(bottom Ekman)}}}} - \underbrace{\boxed{\frac{\beta_0}{f_0^2 \rho} \frac{\partial p}{\partial x}}}_{\substack{\text{vortex stretching} \\ \text{(planetary } \beta\text{-effect)}}} = 0$$

or $$\underbrace{\frac{d}{2H \cancel{\rho} \cancel{|f_0|}} \frac{|f_0|^{\cancel{2}}}{\beta_0} \nabla^2 p}_{\text{relative vorticity}} + \underbrace{\frac{\partial p}{\partial x}}_{\beta\text{-effect}} = \underbrace{\frac{1}{\cancel{\rho} \cancel{f_0} H} \frac{f_0^{\cancel{2}}}{\beta_0} \vec{k} \cdot \nabla \times \vec{\tau}}_{\text{known forcing (curl of wind) vorticity}}$$

or $$\boxed{\frac{d}{2H} \frac{|f_0|}{\beta_0} \nabla^2 p + \frac{\partial p}{\partial x} = \frac{1}{H} \frac{f_0}{\beta_0} \vec{k} \cdot \nabla \times \vec{\tau}}$$

ocean gyre
western boundary current
Stommel + Sverdrup

bottom Ekman layer **beta-effect** **surface Ekman layer**

$\beta = \frac{\beta_0 L}{f_0}$

$$\frac{\text{relative vorticity}}{\beta\text{-effect}} = \frac{\frac{d\,|f_0|\,\nabla^2 p}{2H\beta_0}}{\frac{\partial p}{\partial x}} \sim \frac{d}{H} \cdot \frac{1}{\beta} = \frac{E_v^{1/2}}{\beta} \approx 0.02$$

turbulence-enhanced viscosity

$$\left.\begin{array}{l} A \sim 10^{-2}\ m^2 s^{-1} \\ f_0 \sim 10^{-4}\ s^{-1} \end{array}\right\} \Rightarrow d \sim 15\text{-}m \qquad d = \left(\frac{2A}{|f_0|}\right)^{1/2}$$

$$H \sim 3000\ m \qquad \frac{d}{H} \sim 0.005$$

$$\beta = \frac{\beta_0 L}{f_0}$$

$$\left.\begin{array}{l} \beta_0 \sim 2 \cdot 10^{-11}\ m^{-1} s^{-1} \\ L \sim 10^6\ m \end{array}\right\} \beta \sim 0.2$$

$\Rightarrow$ neglect relative vorticity (from bottom friction; small relative to β-effect)

$$\Rightarrow \quad \frac{\partial p}{\partial x} = \frac{f_0}{\beta_0 H}\left(\frac{\partial \tau^{(y)}}{\partial x} - \frac{\partial \tau^{(x)}}{\partial y}\right)$$

or

neglect this for ease of integration

$$v = \frac{1}{\rho_0 \beta_0 H}\left(\cancel{\frac{\partial \tau^{(y)}}{\partial x}} - \frac{\partial \tau^{(x)}}{\partial y}\right) \qquad \boxed{\text{Sverdrup (1947)}}$$

$$\text{or} \quad p = P_1(y) - \frac{f_0}{\beta_0 H}\int_{L_1}^{x} \frac{\partial \tau^{(x)}}{\partial y}\,dx = P_1(y) - \frac{f_0}{\beta_0 H}\frac{\partial \tau^{(x)}}{\partial y}(x - L_1)$$

constant in x wind-stress curl

and

$$u = -\frac{1}{f_0 \rho}\frac{\partial p}{\partial y} = -\frac{1}{f_0 \rho}\frac{\partial P_1}{\partial y} + \frac{f_0}{f_0 \beta_0 H}\frac{\partial}{\partial y}\int_{L_1}^{x} \frac{\partial \tau^{(x)}}{\partial y}\,dx$$