$\beta = \frac{\beta_0 L}{f_0}$

$$\frac{\text{relative vorticity}}{\beta\text{-effect}} = \frac{\frac{d\,|f_0|\,\nabla^2 p}{2H\beta_0}}{\frac{\partial p}{\partial x}} \sim \frac{d}{H} \cdot \frac{1}{\beta} = \frac{E_v^{1/2}}{\beta} \approx 0.02$$

$d = \left(\frac{2A}{|f_0|}\right)^{1/2}$

turbulence-enhanced viscosity $\left.\begin{array}{l} A \sim 10^{-2}\ m^2 s^{-1} \\ f_0 \sim 10^{-4}\ s^{-1} \end{array}\right\} \Rightarrow d \sim 15\text{-}m$

$H \sim 3000\ m \qquad \frac{d}{H} \sim 0.005$

$\beta = \frac{\beta_0 L}{f_0}$

$\left.\begin{array}{l} \beta_0 \sim 2 \cdot 10^{-11}\ m^{-1} s^{-1} \\ L \sim 10^6\ m \end{array}\right\} \quad \beta \sim 0.2$

$\Rightarrow$

neglect relative vorticity (from bottom friction; small relative to β-effect)

$$\Rightarrow \quad \frac{\partial p}{\partial x} = \frac{f_0}{\beta_0 H}\left(\frac{\partial \tau^{(y)}}{\partial x} - \frac{\partial \tau^{(x)}}{\partial y}\right)$$

<u>simplified vorticity equation</u>
<u>"Sverdrup Balance"</u>

explains ocean gyres

<u>*geostrophic velocity*</u>

or

neglect this for ease of integration

$$v = \frac{1}{\rho_0 \beta_0 H}\left(\cancel{\frac{\partial \tau^{(y)}}{\partial x}} - \frac{\partial \tau^{(x)}}{\partial y}\right)$$

$\boxed{\text{Sverdrup (1947)}}$

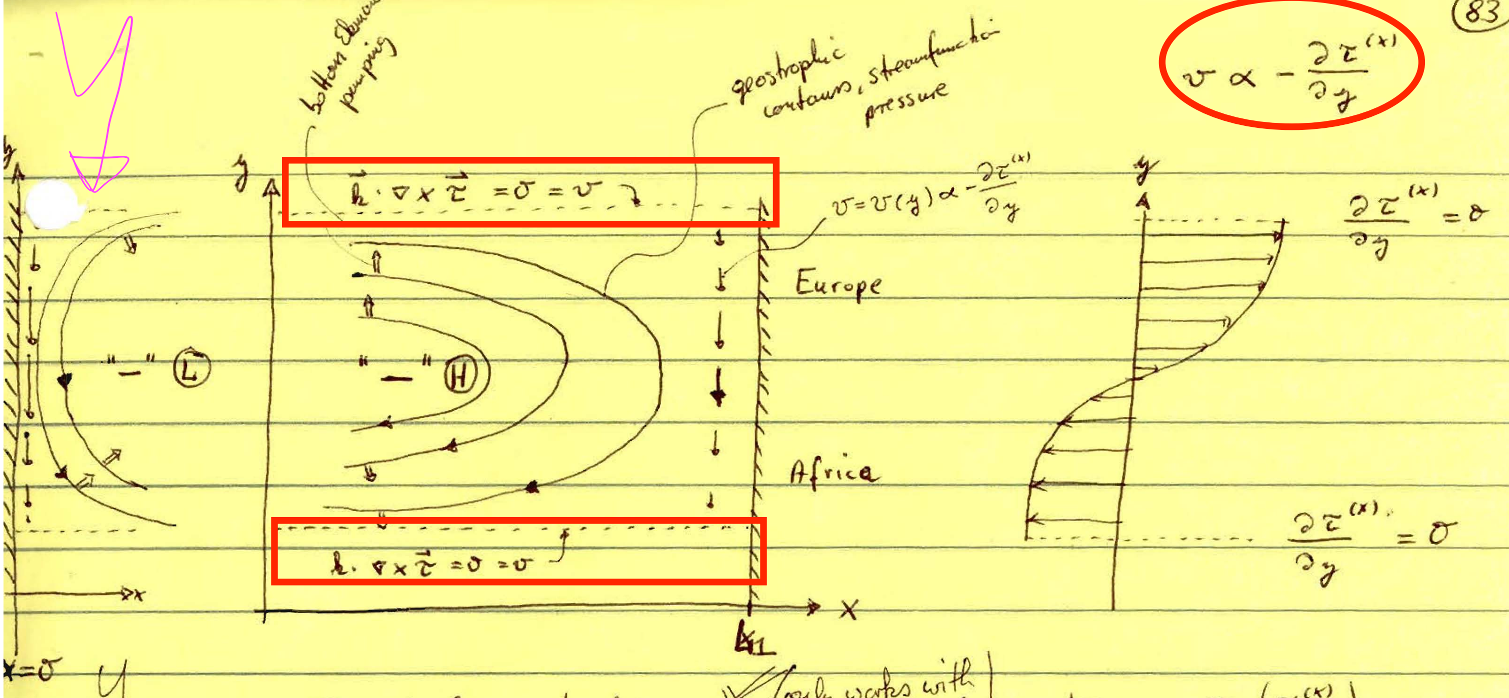
$$\text{or} \quad p = P_1(y) - \frac{f_0}{\beta_0 H}\int_{L_1}^{x} \frac{\partial \tau^{(x)}}{\partial y}\, dx = P_1(y) - \frac{f_0}{\beta_0 H}\frac{\partial \tau^{(x)}}{\partial y}(x - L_1)$$

constant in x wind-stress curl

and

$$u = -\frac{1}{f_0 \rho}\frac{\partial p}{\partial y} = -\frac{1}{f_0 \rho}\frac{\partial P_1}{\partial y} + \frac{f_0}{f_0 \beta_0 H}\frac{\partial}{\partial y}\int_{L_1}^{x}\frac{\partial \tau^{(x)}}{\partial y}\, dx$$

$v \propto -\frac{\partial \tau^{(x)}}{\partial y}$



surface & bottom Ekman flux in the same direction not realizable

This is the geostrophic interior circulation that results from a balance of Ekman pumping from the wind-stress curl

(only works with coast in the East) $w_E^{top} = -\frac{\partial}{\partial y}\left(\frac{\tau^{(x)}}{\rho f_0}\right)$

$\rightarrow \frac{\partial \tau^{(x)}}{\partial y} > 0 \rightarrow \begin{cases} \text{downwelling } f_0 > 0 \\ \text{upwelling } f_0 < 0 \end{cases}$

The downwelling Ekman pumping (northern hemisphere) sets up the interior geostrophic pressure field, however,

we can satisfy only 1 boundary condition for the 1st order PDE
In order to apply other boundary conditions we need the higher order derivative $\nabla^2 p$ reflecting, perhaps, vertical (Stommel, 1948) $\propto \nabla^2 p$ or lateral (Munk, 1950) friction $\propto \nabla^4 p$

Recall our scaling of

$$\frac{\text{relative vorticity}}{\beta\text{-effect}} \sim \frac{d}{H} \cdot \frac{f_0}{\beta_0 L} \approx 0.02 \quad \text{for } L \sim 10^6\,\text{m}$$

At such small (horizontal) boundary layer scales the wind stress curl is negligible. ≈ 1 for $L \sim 20\,\text{km}$