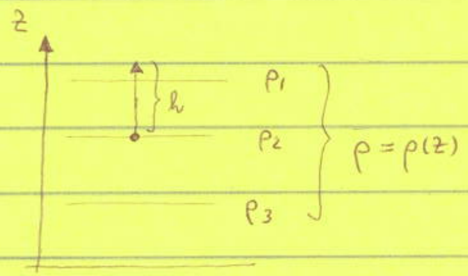
Stratification

so far $\rho' = 0$ & $p_{static} = p_a + \rho_0 g z$



incompressible $\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \vec{u} \cdot \nabla \rho = 0$

at the new level the parcel feels a force

Lagrangian statement regarding a particle at location h=h(t)

$$g\left[\rho(z+h) - \rho(z)\right] V$$

$$\frac{m}{s^2} \quad \frac{kg}{m^3} \cdot m^3 = \text{mass} * \text{acceleration} = \text{force}$$

the acceleration of the particle due to this buoyancy force is

$$\boxed{\boxed{\rho(z)} V \cdot \frac{d^2 h}{dt^2} = g \boxed{\left[\rho(z+h) - \rho(z)\right]} \cdot V}$$

but changes in density are small relative to a reference density
(Boussinesq approximation)

first term in a Taylor expansion

& $\boxed{\rho(z) \longrightarrow \rho_0}$ and $\boxed{\rho(z+h) - \rho(z) \longrightarrow \frac{\partial \rho}{\partial z} \cdot h}$

and we get

$$\rho_0 \cdot \cancel{V} \frac{d^2 h}{dt^2} \simeq g \frac{\partial \rho}{\partial z} \cdot h \cdot \cancel{V}$$

or

$$\boxed{\frac{d^2 h}{dt} - \frac{g h}{\rho_0} \frac{\partial \rho}{\partial z} = 0}$$

or better yet

$$\boxed{\frac{d^2 h}{dt^2} + N^2 h = 0} \qquad N = \sqrt{\frac{-g}{\rho_0}\frac{\partial \rho}{\partial z}}$$

Brunt-Vaisala Frequency

Buoyancy Frequency

case-1: $\frac{\partial \rho}{\partial z} < 0$ N is real, $N^2 > 0$ → $h(t) = A\cos(N \cdot t)$ harmonic oscillation (stable stratification)

case-2: $\frac{\partial \rho}{\partial z} > 0$ N is imaginary, $N^2 < 0$ → $h(t) = A e^{Nt} \xrightarrow[t \to \infty]{} \infty$ convection (unstable distribution)

Same argument applies for (compressible) atmosphere if we interpret

review gas law

C_p ✓ (done, see p.93-9

$$N^2 = \frac{g}{T}\left(\frac{dT}{dz} + \underbrace{\frac{g}{C_p}}_{}\right)$$

dry adiabatic lapse rate $\approx \frac{10°C}{km}$ atmosphere (cooler on top of mountain where pressure is lower)

$\frac{0.1°C}{km}$ ocean

→ introduce "potential ~~density~~ temperature" θ defined as the temperature that a parcel would have if it were brought adiabatically to a given reference pressure, then

$$N = \sqrt{\frac{g}{\theta}\frac{d\theta}{dz}}$$

this allows us a treat the compressible fluid as if it were incompressible