Hierarchy of time scales:

Rotational time scale (1/f) much smaller than advective time scal
Advective time scale (L/U) much smaller than diffusive time scale

# QG Time Scales

(a) consider frictional time scale $\Delta t_d$

$$\frac{\partial q}{\partial t} \sim \nu \frac{\partial^2 \xi}{\partial z^2} \qquad \text{diffusion of relative vorticity}$$

$$\text{or} \quad \frac{\partial \xi}{\partial t} \sim \nu \frac{\partial^2 \xi}{\partial z^2} \qquad \text{— " —}$$

$$\text{or} \quad \frac{\xi}{\Delta t_d} \sim \frac{\nu \xi}{H^2} \longrightarrow \Delta t_d \sim \frac{H^2}{\nu} \longrightarrow \frac{\Delta t_d}{1/f} \sim \frac{f H^2}{\nu} = E_v^{-1}$$

$$\approx 10^4$$

10,000 inertial periods 1/f

(b) consider advective time scale $\Delta t_a$

$$\frac{\partial q}{\partial t} \sim J(\psi, q) = \underline{u_o \frac{\partial q}{\partial x} + v_o \frac{\partial q}{\partial y}}$$

$$\text{or} \quad \frac{q}{\Delta t_a} \sim \underline{\frac{U q}{L}} \longrightarrow \Delta t_a \sim L/U \longrightarrow \frac{\Delta t_a}{1/f} \sim \frac{L \cdot f}{U} \sim R_o^{-1}$$

$$\approx 10^2$$

100 inertial periods 1/f

$$\text{Thus} \quad \frac{\Delta t_a}{\Delta t_d} \sim \frac{E_v}{R_o} \ll 1$$

so the advective time scale dominates and for QG dynamics the interior flow is NOT diffusive

$$\boxed{\text{Note} \quad 1/f \ll \Delta t_a \ll \Delta t_d}$$

# Linear QG Rossby Waves with Stratification

(1) Nonlinear QG Potential Vorticity

(2) Ignore frictional dissipation $\Delta t_d \gg 1/\omega$

(3) set $N = \text{const.}$

(4) Linearize PV equation, that is,

neglected all nonlinear terms

$$J(\psi, q) = J(\psi, \underbrace{\nabla^2\psi + \frac{f_0^2}{N^2}\frac{\partial^2\psi}{\partial z^2} + \beta_0 y}_{q}) \approx J(\psi, \beta_0 y)$$

$$= \frac{\partial\psi}{\partial x}\cdot\beta_0$$

$$J(\psi, q) \approx \frac{\partial\psi}{\partial x}\beta_0$$

requires

$\longrightarrow$ (a) $\boxed{\xi \equiv \nabla^2\psi \ll \beta_0 y}$

or $\frac{U}{L} \ll \beta_0 L \rightarrow \frac{U}{f_0 L} \ll \frac{\beta_0 L}{f_0}$

or $\boxed{R_0 \ll \beta}$ $\qquad \beta \equiv \frac{\beta_0 L}{f_0}$

$\longrightarrow$ (b) $\boxed{\frac{f_0^2}{N^2}\frac{\partial^2\psi}{\partial z^2} \ll \beta_0 y}$

or $$\frac{f_0^2}{N^2}\frac{UL}{H^2} \ll \beta_0 L \longrightarrow U \ll \beta_0 \left(\frac{L_D}{L}\right)^2$$

or $$\frac{U}{f_0 L} \ll \frac{\beta_0 L}{f_0}\left(\frac{L_D}{L}\right)^2$$

or $$\boxed{R_o \ll \beta \cdot B_u}$$

$$B_u = \frac{(NH)^2}{(f_0 L)^2} = \left(\frac{L_D}{L}\right)^2 \hat{=} \frac{\text{rel. vertical}}{\text{stretching}}$$

(c) Also required in the density equation that

$$\frac{\partial \rho'}{\partial t} + \underbrace{J(\psi, \rho')}_{\text{nonlinear}} - \underbrace{\frac{\rho_0 N^2}{g} w_1}_{\text{linear term requires } \psi(x,y,z)} = 0$$

$$\boxed{\frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial z}\right) \gg J\left(\psi, \frac{\partial \psi}{\partial z}\right)}$$

and $\rho' = -\frac{1}{g}\frac{\partial p_0}{\partial z} = \frac{\rho_0 f_0}{g}\frac{\partial \psi}{\partial z}$

or $$\frac{\partial \rho'}{\partial t} \gg J(\psi, \rho') = \frac{\partial \psi}{\partial x}\frac{\partial \rho'}{\partial y} - \frac{\partial \psi}{\partial y}\frac{\partial \rho'}{\partial y} = v_0 \frac{\partial \rho'}{\partial y} + u_0 \frac{\partial \rho'}{\partial x}$$

$$\frac{\rho'}{\Delta t} \gg \frac{U \rho'}{L} \quad ..$$

$$R_{o_T} = \frac{1/f_0}{\Delta t} \gg \frac{U}{f_0 L} = R_o$$

or $$1 \gg R_{o_T} \gg R_o$$

or $$\boxed{F_r \equiv \frac{U}{C} = \frac{R_o}{R_{o_T}} \ll 1}$$