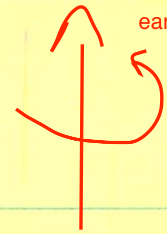


earth rotational vector

at North Pole the normal component of the earth's rotational vector is f=+fmax

Equator normal component of the earth's rotational vector is f=0

at South pole the normal component of earth's rotational vector is f=-fmax

10+

Rossby Waves

Vorticity Waves due to (a) $f = f(y) = f_0 + \beta y$ Coriolis varies with latitude

beta-plane aproximation

(b) $D \neq D(x, y)$ Depth varies

Consider shallow water equations

$$\boxed{\begin{aligned} &(1)\quad u_t - f v = -g\eta_x \\ &(2)\quad v_t + f u = -g\eta_y \end{aligned}}$$

$\Big| \cdot \frac{\partial}{\partial y}$ $\quad u_{ty} - \beta v - f v_y = -g\eta_{xy}$

$\Big| \cdot \frac{\partial}{\partial x}$ $\quad v_{tx} + f u_x = -g\eta_{yx}$

Take the curl of the momentum to derive the vorticity equation [just for the vertical component of the vorticity vector].

(2)-(1):

$$\boxed{(v_x - u_y)_t + f(u_x + v_y) + \beta v = 0}$$

From continuity

$$\eta_t + (uD)_x + (vD)_y = 0$$

with ~~$D(y) = e^{-By/f}$~~ $D(y) = e^{-By/f}$ this continuity gives

$$(u_x + v_y) = -\frac{\eta_t}{D} - \frac{v}{D}(-)\frac{B}{f} D$$

and the vorticity equation becomes

$$\boxed{(v_x - u_y)_t = -\beta v - B v + \frac{f}{D}\eta_t}$$

The "planetary β" has the same dynamical as the "topographic B" effect!

$$\eta_t + (uD)_x + (vD)_y = 0$$ Continuity

$$\eta_t + u_x \cdot D + u D_x + v_y D + v D_y = 0$$

$$\eta_t + D(u_x + v_y) + \vec{u} \cdot \vec{\nabla} D = 0$$

$$(u_x + v_y) = \frac{-\eta_t}{D} - \frac{\vec{u}}{D} \nabla D$$

into (2)-(1)

$$(v_x - u_y)_t + \frac{f}{D}(-)\eta_t + \frac{f}{D}(-)\vec{u}\nabla D + \beta v = 0$$

with $D = D(y) = e^{-By/f}$ $\qquad D_y = -\frac{B}{f} e^{-By/f}$

$$(v_x - u_y)_t - \frac{f}{D}\eta_t - \frac{f}{\cancel{D}} v (-)\frac{B}{\cancel{f}} \cdot \cancel{D} + \beta v$$

$$(v_x - u_y)_t - \frac{f}{D}\eta_t + Bv + \beta v = 0$$

Let focus on "planetary-β"

D = const.

horizontally non-divergent ocean, e.g., $u_x + v_y = 0$ $\quad (\eta_t = 0)$

[very low-frequency $\eta_t \to 0$]

D/T << D U/L or
f << U/L or
f*U/L << 1 or
Rossby Number << 1

The vorticity equation then becomes

$$\boxed{\underbrace{(v_x - u_y)_t}_{\text{relative vorticity}} + \underbrace{\beta v}_{\text{planetary vorticity}} = 0}$$

non-divergent flow $\Rightarrow$ $u = -\psi_y$ and $v = \psi_x$ stream function

$$\boxed{\nabla^2 \psi_t + \beta \psi_x = 0}$$

What does this say about the relation of velocity (u,v) and wave number vector (k,l)?

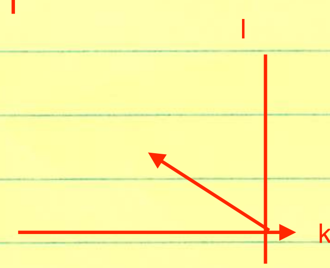
u —> -i*l
v —> +i*k
|u/v| = | l/k |

Plane wave solutions

$$\psi = e^{-i\sigma t + ikx + ily}$$

give dispersion

$$\boxed{\sigma = \frac{-\beta k}{k^2 + l^2}}$$



which we can write as

$$\boxed{\left(k + \frac{\beta}{2\sigma}\right)^2 + l^2 = \left(\frac{\beta}{2\sigma}\right)^2}$$

which in the (k, l) plane looks like

$$\left(k + \frac{\beta}{2\sigma}\right)^2 + l^2 = \left(\frac{\beta}{2\sigma}\right)^2$$

$$k^2 + 2\frac{k\beta}{2\sigma} + \cancel{\left(\frac{\beta}{2\sigma}\right)^2} + l^2 = \cancel{\left(\frac{\beta}{2\sigma}\right)^2}$$

$$k^2 + l^2 = -k\frac{\beta}{\sigma}$$

$$\boxed{\sigma = -k\frac{\beta}{k^2+l^2}}$$

$$k^2 + l^2 = -k\frac{\beta}{\sigma}$$

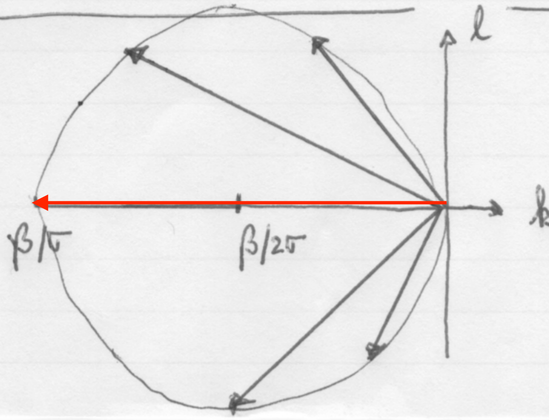
$$k^2 + l^2 + 2k\frac{\beta}{2\sigma} + \left(\frac{\beta}{2\sigma}\right)^2 - \left(\frac{\beta}{2\sigma}\right)^2 = 0$$

$$k^2 + 2k\frac{\beta}{2\sigma} + \left(\frac{\beta}{2\sigma}\right)^2 + l^2 = \left(\frac{\beta}{2\sigma}\right)^2$$

$$\boxed{\underline{\left(k + \frac{\beta}{2\sigma}\right)^2} + \underline{l^2} = \underline{\left(\frac{\beta}{2\sigma}\right)^2}}$$

$$\boxed{a^2 + b^2 = c^2}$$

which wave numbers (k, l) are on this circle



That's a circle of radius $\beta/2\sigma$

displaced on the k-axis by $\beta/2\sigma$ to the left!