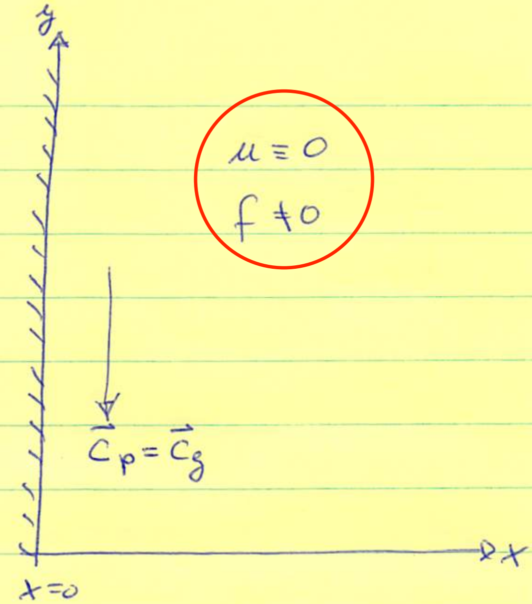
5.8 (p.121) Kelvin Wave

Solid Wall $x = 0$, $u \equiv 0$

$u \equiv 0$

$f \neq 0$



$-fv = -g\eta_x$ geostrophic across-shore

$v_t = -g\eta_y \quad \Big| \frac{\partial}{\partial y}$

$\eta_t + D v_y = 0 \quad \Big| \frac{\partial}{\partial t}$

$$\eta_{tt} - gD\,\eta_{yy} = 0$$

pressure gradient along wall due to sealevel oscillations produce an acceleration along the wall, but pressure gradient along wall is in geostrophic balance with along-channel flow

$e^{-i\sigma t}$

$-\sigma^2 - gD\,\eta_{yy} = 0$

$$\eta_{yy} + \frac{\sigma^2}{gD} = 0$$

choose

$$\eta = a(x)\, e^{ily}$$

where $a(x)$ still unknown

$\Downarrow \quad \sigma^2 = gD\, l^2$ same as shallow water wave without rotation

Kelvin Wave Dispersion Eq.

Combine $v = \frac{g}{f}\eta_x$ and $-i\sigma v = -g\eta_y$ from momentum

$$\frac{g}{f}\eta_x = \frac{g}{i\sigma}\eta_y \quad \text{or} \quad -i\sigma\,\eta_x + f\,\eta_y = 0$$

$$\eta = a(x) \cdot e^{ily}$$

$$-i\sigma \eta_x + f\eta_y = 0$$

$$-i\sigma \frac{da}{dx} e^{ily} + f i l e^{ily} = 0$$

ordinary differential equation for a(x):

$$\frac{da}{dx} = \frac{f}{\sigma} l \quad \text{or} \quad a(x) = a_0 e^{flx/\sigma}$$

l/sigma = 1/c_phase
l/sigma = 1/sqrt(gD)

And the full solution is

$$\eta = a_0 e^{-i\sigma t + ily} e^{flx/\sigma}$$

Waves for $x > 0$

- finite solution as $x \to \infty$
- $\lim_{x \to \infty} \eta = 0$
- $l < 0$
- wave travels in the $-y$ direction here
- for $f > 0$ wave always travels with coast on its right