



# **Subinertial Oscillations on the Amundsen Sea Shelf, Antarctica**

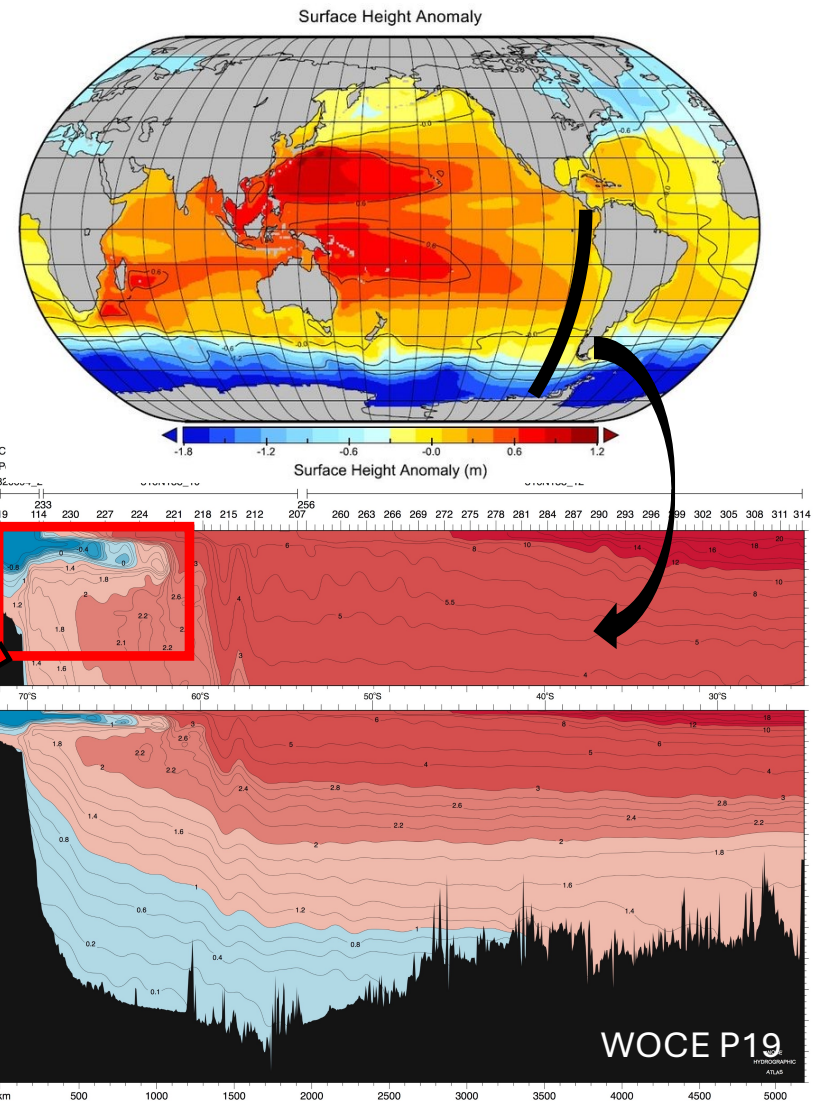
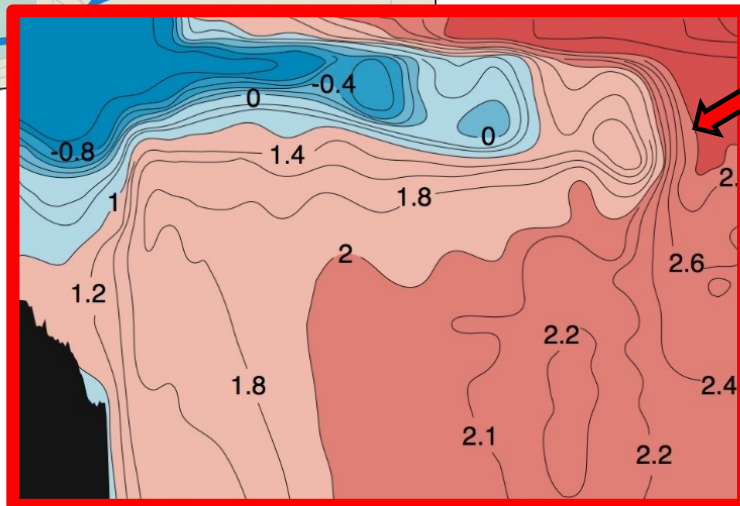
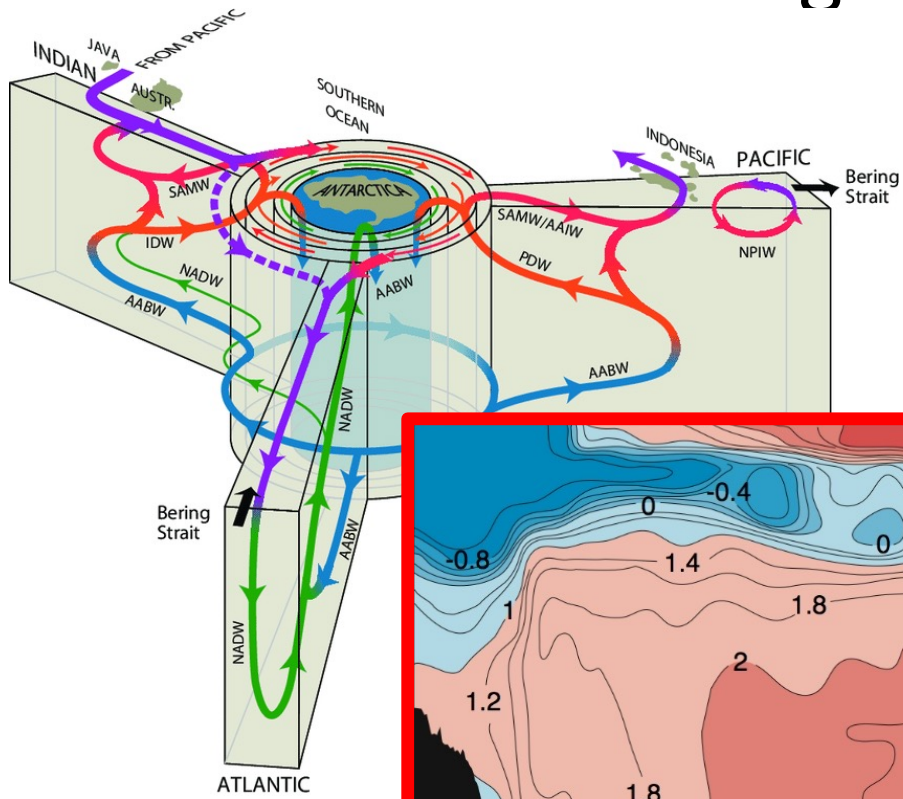
**Wahlin et al. 2016**

**Presented by Michael Cappola**

**MAST 655**

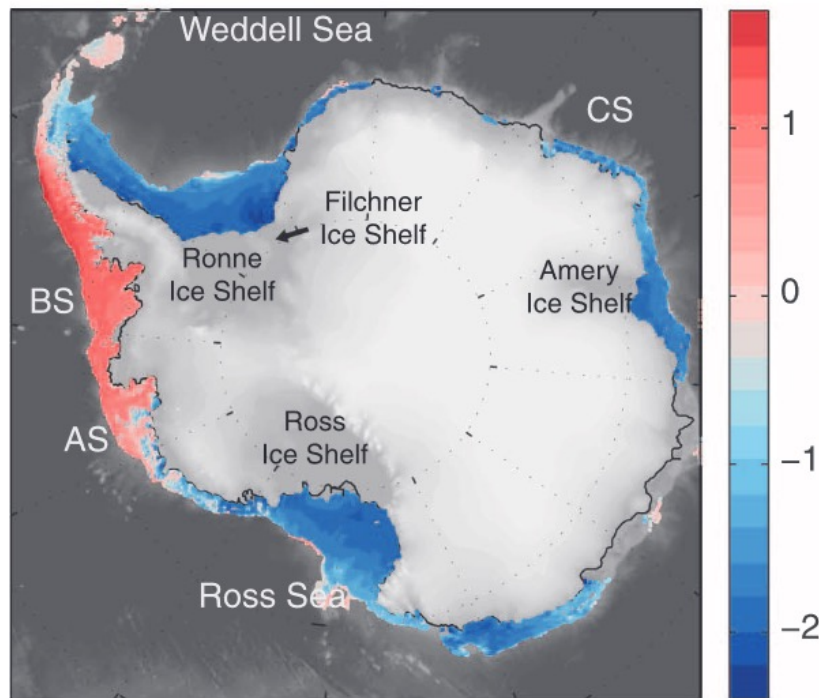
**20240506**

# Antarctic Oceanography

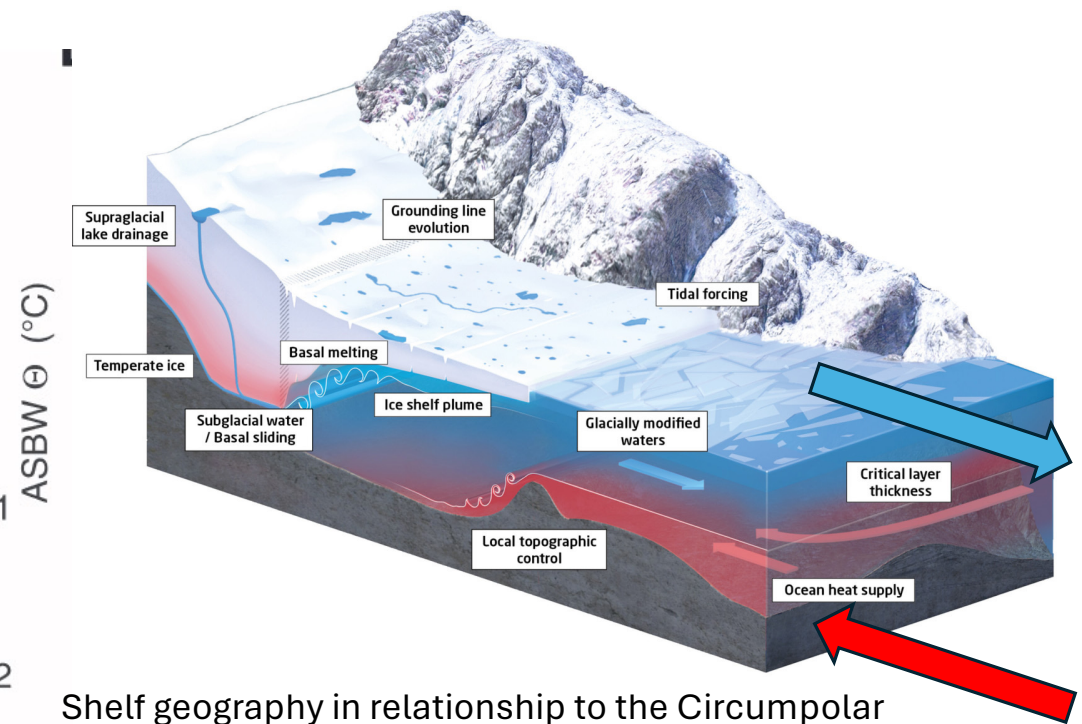




# Antarctic Oceanography



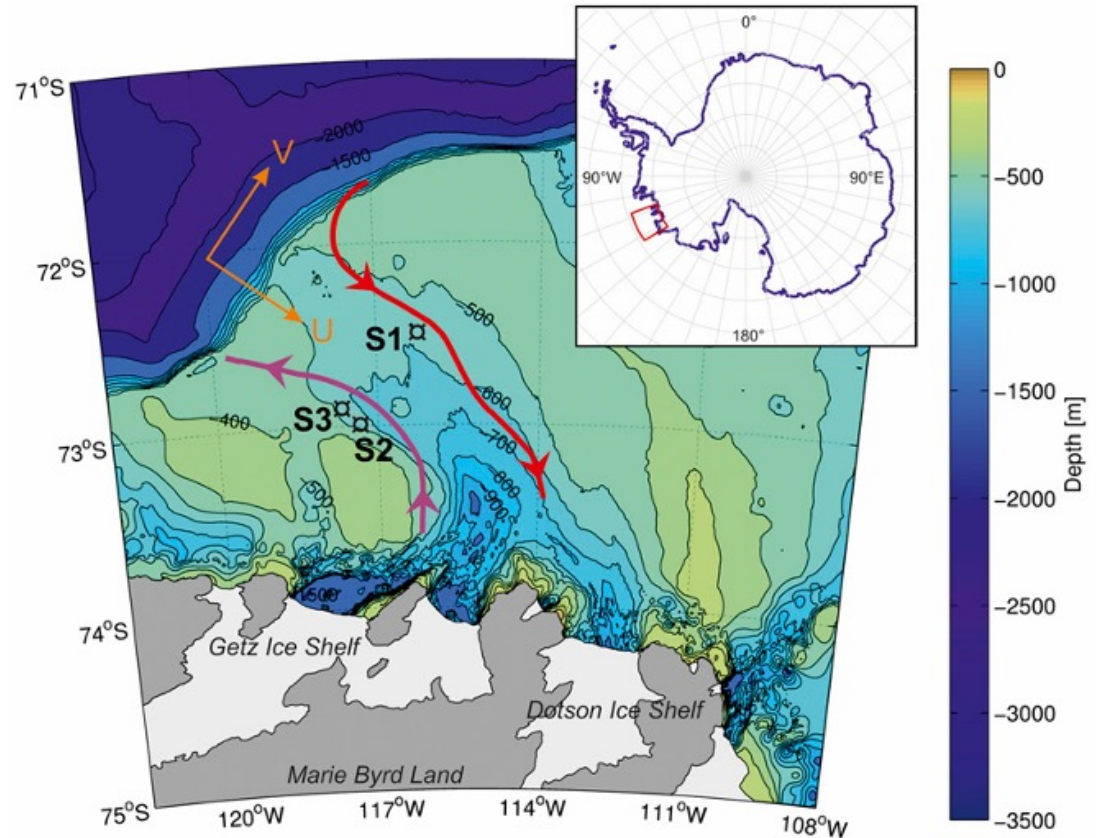
Schmidtko, S., Heywood, K. J., Thompson, A. F., & Aoki, S. (2014). Multidecadal warming of Antarctic waters. *Science*, 346(6214), 1227–1231. <https://doi.org/10.1126/science.1256117>



Shelf geography in relationship to the Circumpolar Current direction, as well as residence time, dictates whether a shelf is “warm” or “cold”

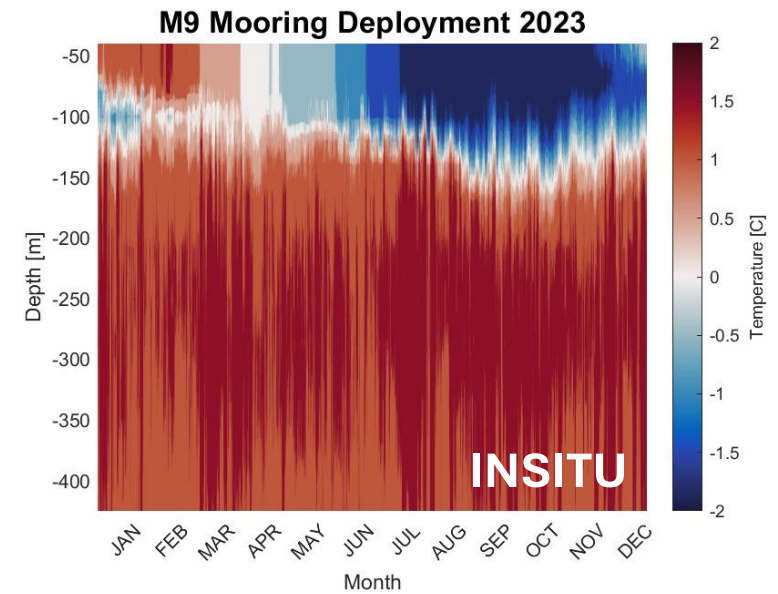
# Amundsen Sea Shelf

- Dotson Trough
  - Warm Water Inflow
    - 0.75 C
    - 34.6 PSU
  - Cold Water Outflow
    - 1.25 C colder than inflow
    - 0.3 PSU fresher than inflow
  - Net Estimated Melt
    - Budget calculations suggest 80-240 km<sup>3</sup> / y of glacial melt
    - Subject of Ha et al 2014
- Moorings
  - Data source
  - Placed in the inflow and outflow from 2010 – 2014, each for a 2- year deployment.
  - **What are moorings?**



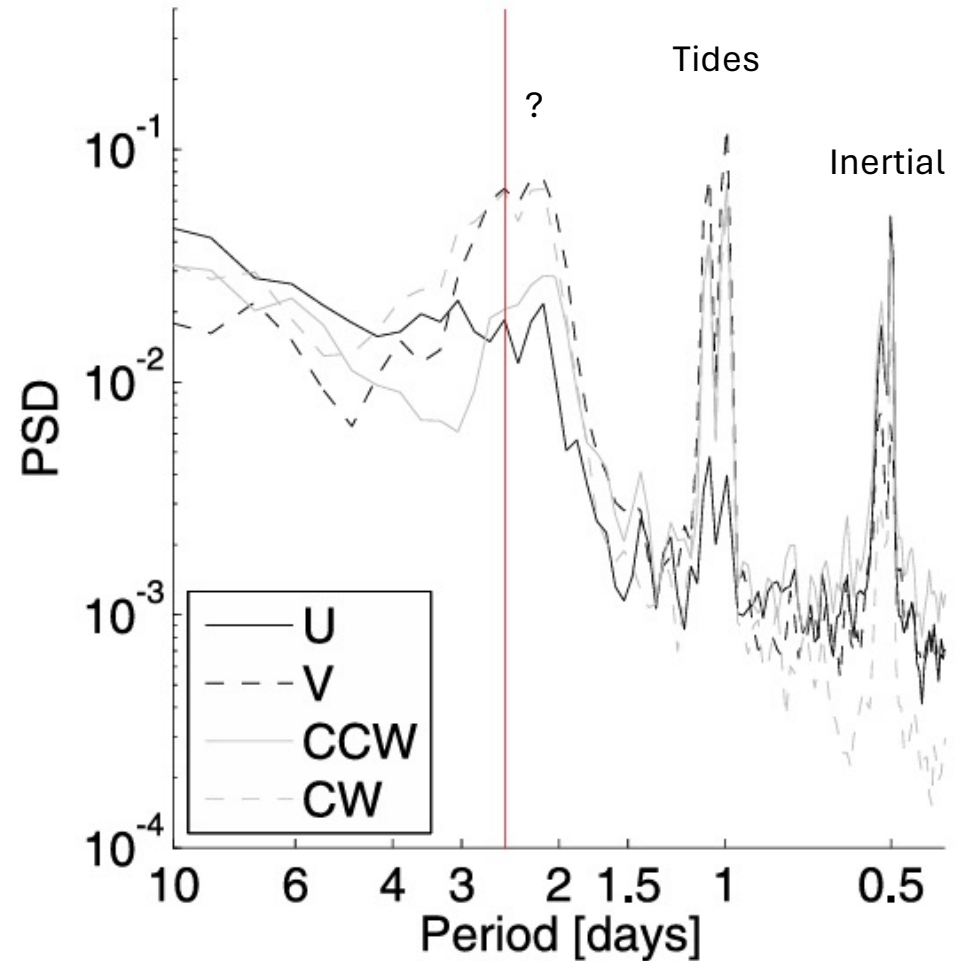
# Moorings

- Eulerian
- Sensor timeseries can be vertically interpolated
- Gives a representation of the ocean
  - **Depth vs Time**
- **Microcat = Temp, Salinity, Pressure**
- **ADCP = Current Profile**
  - **Acoustic Doppler Current Profiler**
- Data processing procedure
  - Download
  - **De-tide**
  - Transform coordinate system for velocity
    - Rotate V and U
- De-tiding involves using a Fourier Transform to convert the data from the **time domain** to the **frequency domain**
- Tides of a known frequency can then be removed



# Unknown Signal in S2

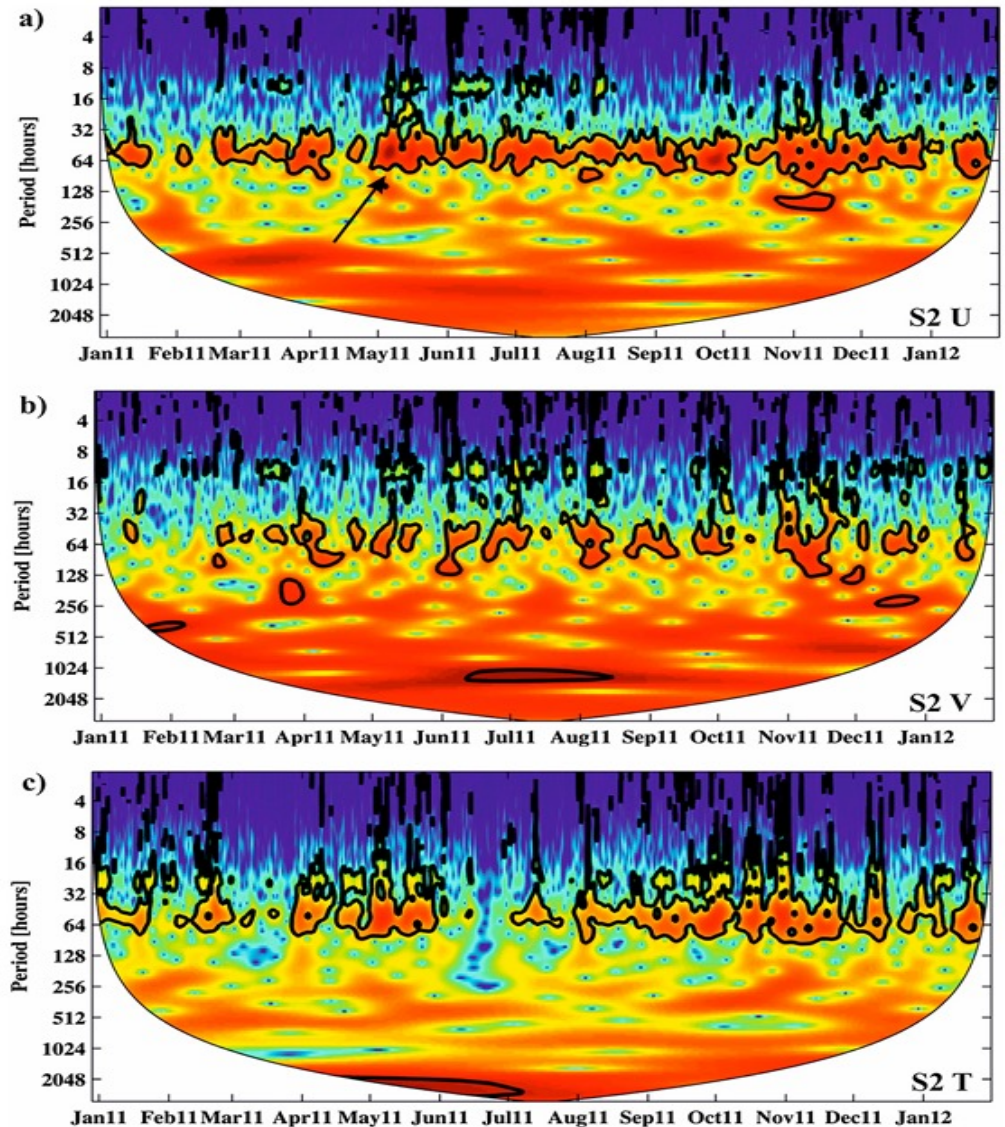
- Fourier Spectra of Velocity 100m above bottom
  - Power Spectral Density (amplitude) vs Frequency
- Near Daily Tidal Signals
- Inertial Signal
  - $T = (2\pi)/f(\text{latitude}) = 12.514$  hours
- Unknown signal
  - 40–80-hour periodicity
  - **Strongest in V (across-channel)**
  - Also seen in temperature and salinity spectra.
  - Similar magnitude as the tides!
- The same signal being present in all data sources implies that the water column is moving over the mooring, and we are seeing an oscillation in the **layers** as opposed to an oscillating velocity **within the layer**.





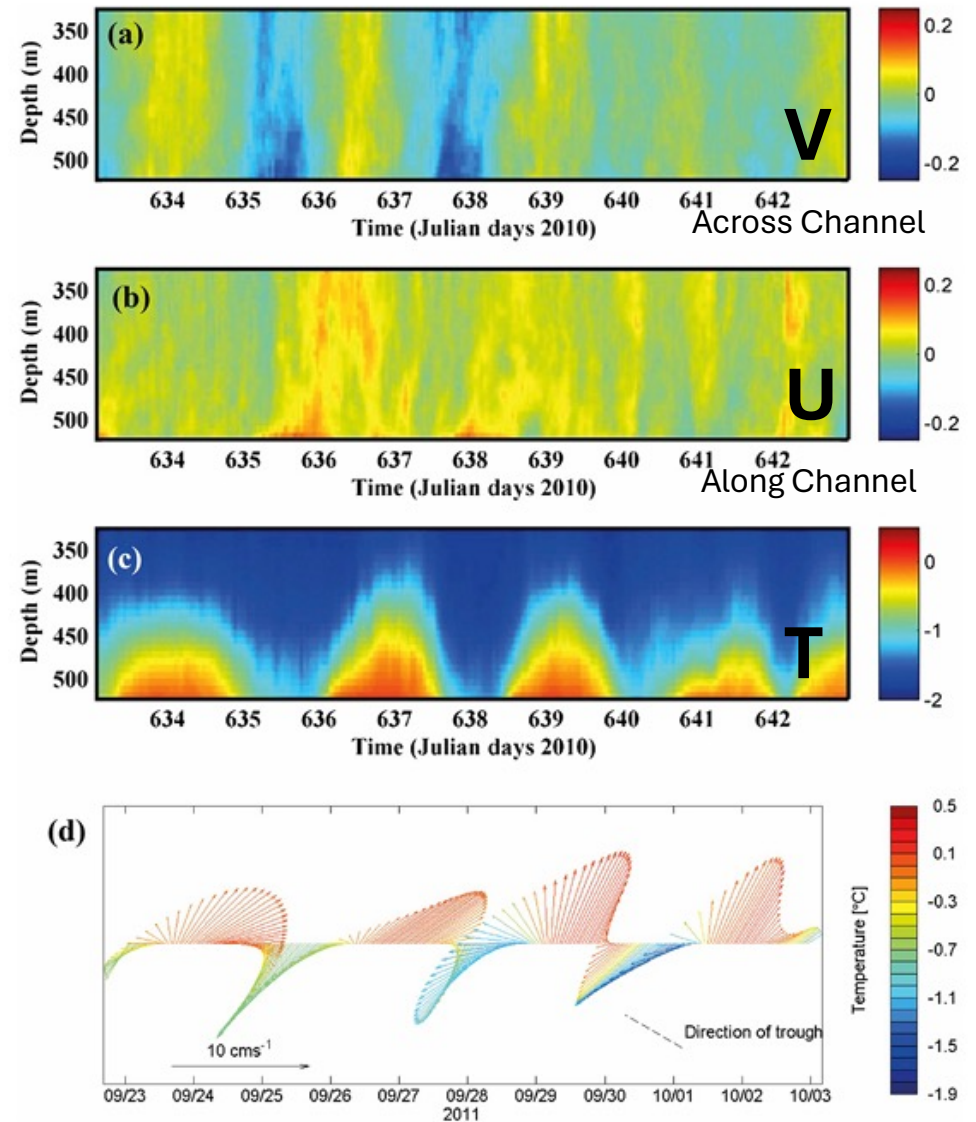
# Wavelet Analysis

- Wavelet Analysis
  - Vertically Averaged U and V
  - Bottom Temperature
- Shows that mystery signal has little temporal variation
  - 40–80-hour signal is always there
  - **No seasonality**
  - **No frequency drift**
- 95% Confidence = Contour Line
- Wavelets keep the time component while still representing frequency power spectra.
  - Allows you to see:
    - Time oscillations in frequency space.
    - Frequency drift with respect to time.



# Back to Time Domain

- In situ data during one oscillation peak
  - Temp and velocity co-oscillate
  - Warm Water = Northeast Velocity
  - Cold Water = Southwest Velocity
- Interpretation:
  - Warm water layer close to the bottom is being moved up and down the slope by strong velocity oscillations.
- **Oscillations are present in the whole water column below 330m.**
- **Oscillations have similar magnitude in both spatial directions.**
- **Oscillations are always present.**
- Is this some sort of resonant wave?



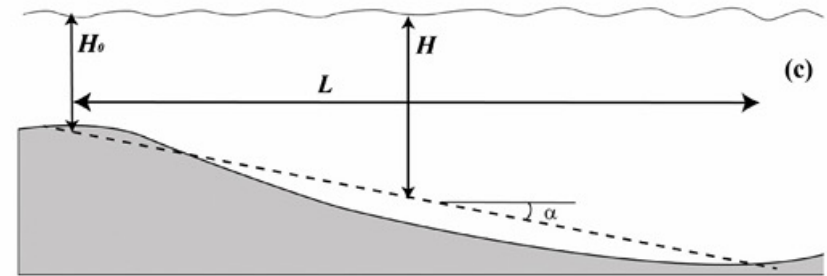


# Topographical Rossby Wave

- Recall:
  - Rossby Waves are slow planetary waves that emerge from the beta-effect.
  - This is a manifestation of the conservation of potential vorticity, and planetary vorticity having a gradient.
    - As the fluid parcel is displaced northward (northern hemisphere), it gains planetary vorticity, so its relative vorticity (spin) must compensate. It gains a clockwise spin. Converse is true for fluid parcels displaced south. The relative vorticity is, in a sense, steering the flow.
    - The net effect of this is a waveform that only travels west (Rossby Wave).
  - Relative vorticity of a displaced fluid is also affected by a gently sloping bottom via vortex tube stretching/squeezing. Displacement to deeper water generates stretching, which changes the relative vorticity. Converse is true for a displacement to shallower water. Net effect is the same.
- Key difference between these is the impacts of stratification. Strong stratification will constrain Topographical Rossby waves to the bottom layer. The beta effect (normal Rossby Wave) holds for the whole water column.

# Analytical Setup

- A simple slope is set up, where  $h$  is a function of  $y$ . Alpha is the slope angle.
- This is added to the continuity equation.  
**Notice that  $f$  is a constant.**
- Finding the vorticity equation, applying a stream function, and assuming the solution of a plane wave yields a dispersion relationship.
- $\beta = (f * \text{Alpha})/H$
- $H = H_0 - (\text{Alpha} * L)/2$ 
  - (average depth of slope)



$$h(y) = H_0 - \alpha y,$$

$$\frac{\partial u}{\partial t} - fv = -g \frac{\partial \eta}{\partial x},$$

$$\frac{\partial v}{\partial t} + fu = -g \frac{\partial \eta}{\partial y},$$

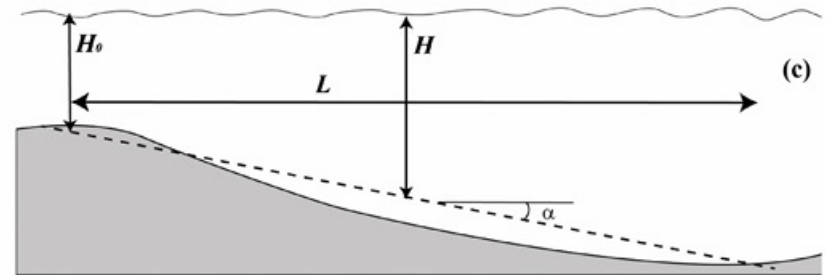
$$\frac{\partial \eta}{\partial t} + H_0 \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \alpha_0 v = 0.$$

$$\Psi = \Psi_0 \sin(kx + ly - \omega t),$$

$$\omega = -\frac{\beta k}{k^2 + l^2 + \frac{f^2}{gH}},$$

# Analytical Setup

- Simple sketch is in one dimension (across channel) so boundary conditions for  $y$  direction are chosen.
  - No velocity at shallow side ( $y=0$ )
  - No change in velocity at deep side ( $y=L$ )
- Dispersion relationship and group velocity can be further simplified with a scale analysis giving:



$$v = 0 \quad \text{at} \quad y = 0$$

$$\frac{\partial v}{\partial y} = 0 \quad \text{at} \quad y = L,$$

$$k^2 \gg f^2 / (gH)$$

$$c_X = \frac{d\omega}{dk},$$

$$\omega \simeq -\frac{\beta k}{k^2 + l^2},$$

$$c_X \simeq \frac{\beta(k^2 - l^2)}{(k^2 + l^2)^2}.$$



# Analytical Setup

- Finally, setting  $k=l$  we see that this gives a group velocity of zero. Solutions will give resonant frequencies.
- Considering the one-dimensional slope and boundary conditions in the  $y$  direction, this limits the wave numbers that can be valid solutions. Valid solutions are given by this expression for  $l$ , for  $n$  modes.
  - Authors considered the first two modes.
- Solutions to these **modes** are eigenmodes and its frequency is an eigenvalue.
- Read about continental shelf wave solutions in Gill 1982 if curious.

$$\omega \approx -\frac{\beta k}{k^2 + l^2},$$
$$c_x \approx \frac{\beta(k^2 - l^2)}{(k^2 + l^2)^2}.$$

$$l_n = \left(n + \frac{1}{2}\right) \frac{\pi}{L},$$

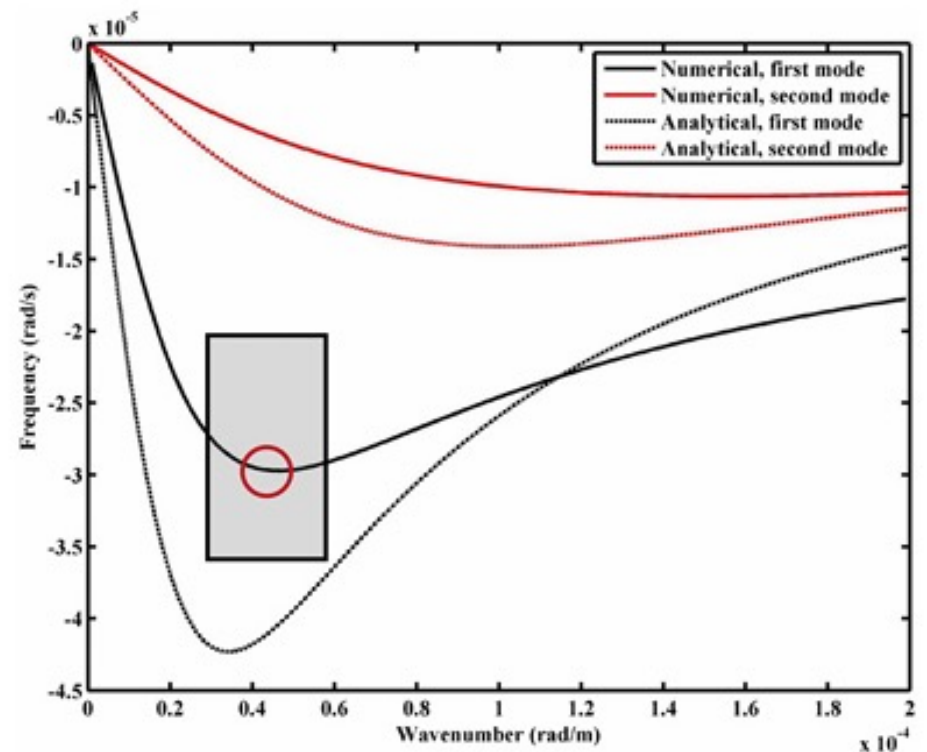
$$\omega_R \approx \frac{\beta}{2l_n}, \quad \text{or} \quad \omega_R \approx \frac{\beta L}{\pi(2n + 1)},$$

Remember, beta is **not** the beta affect  
It's the equivalent term for the shelf slope.

$$\text{Beta} = (f * \text{Alpha})/H$$

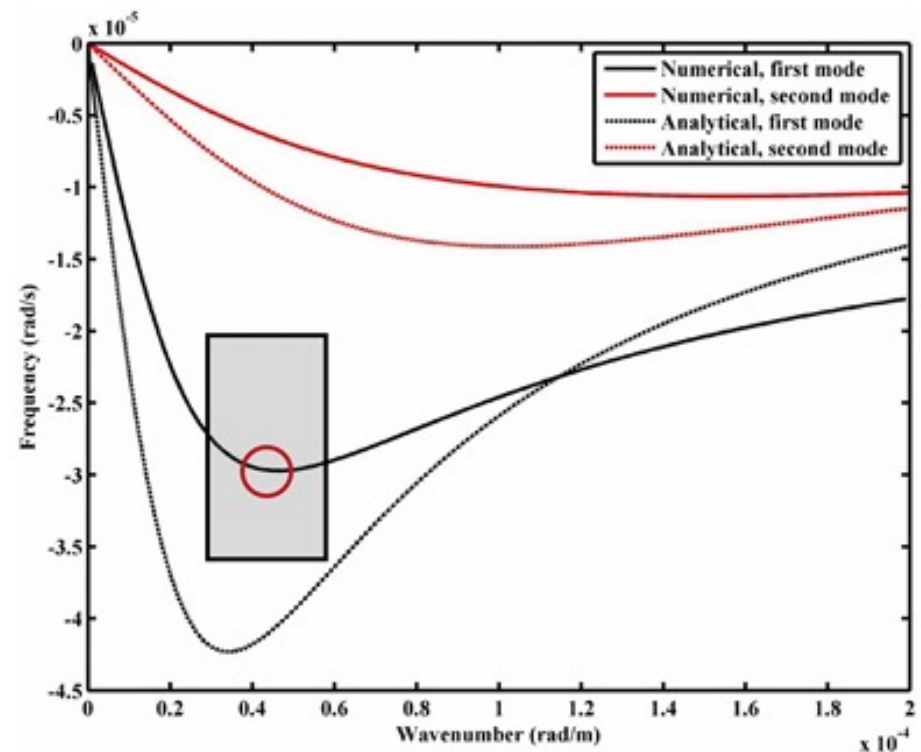
# Results, Analytical and Numerical

- Dispersion Relation
  - Analysis shown on previous slides was calculated numerically using actual bathymetry data and stratification data.
- Black lines are the first eigenmode solutions.
- The y axis limits of the grey box is the 40–80-hour frequency range.
- The red circle in the box is the wave number and frequency for a resonant wave with zero group velocity.



# Results, Analytical and Numerical

- Author used ERA-Interim data for wind and tested the coherence of wind to the vertically averaged velocity oscillations
- **Maximum Coherence (0.69) was found for 67-hour frequency and the wind angle was close to the cross-shelf direction (V+)**
- This frequency also lines up nicely with the red circle for a resonant frequency solution.





# Stratification?

- With the numerical model, they tested the results using measured stratification for the profile.
- This made no change to the solution
- Expected though;  $Bu < 0.1$
- **Bottom slope is a larger contributor than stratification**

- Burger Number (Bu)

$$\frac{\text{Relative vorticity}}{\text{Vertical stretching}} \sim \frac{N^2 H^2}{f_0^2 L^2} = Bu,$$

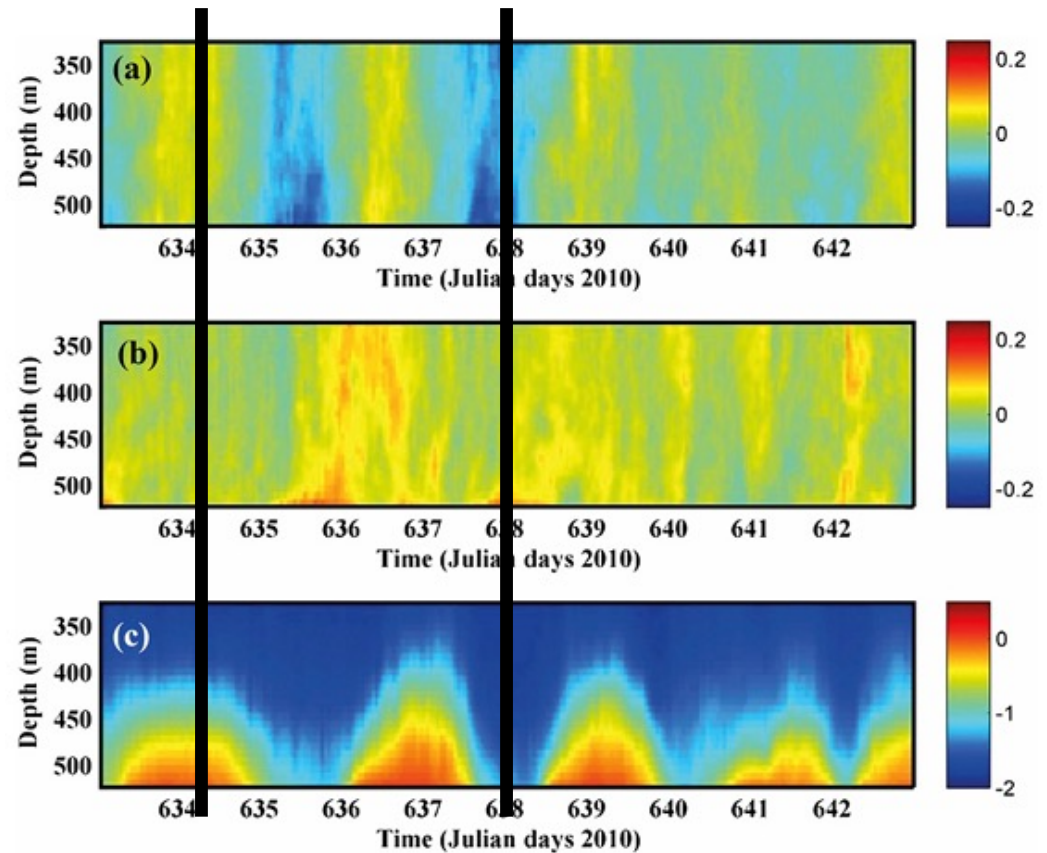
- $Bu \ll 1$ 
  - Weak stratification or long length scale
  - Vertical stretching dominates
  - ~Homogenous geostrophic flow
  - **Motion is heavily impacted by bottom slope**
- $Bu \gg 1$ 
  - Strong stratification or short length scale.
  - Relative vorticity (of each parcel) dominates.
  - This decouples flow into multiple layers that can act independent of each other.
  - **Each parcel obeys its own relative vorticity**

# The Big Idea and Author Conclusion

- In Antarctica, storms are typically on the order of days.
- The high coherence of wind and across shelf velocity suggests that the oscillations observed are resonant topographical Rossby waves driven by wind.
- The wind pushes the water mass slightly down slope (across channel). The parcel moves down slope and stretches vertically. This increases relative vorticity. Conservation of potential vorticity drives oscillations as a topographical Rossby wave.
- This wave physically displaces the layers of the water column, causing velocity, temperature, and salinity, to co-oscillate.
- This same signal was also seen on S3, but at a lower frequency. It has a gentler slope. It was not seen on S1, but the slope would give a periodicity of 157 hours. Weather is not that consistent in antarctica, and it is unlikely that resonance could build at that frequency.

# Implications

- Recall that this is a wave, so streamlines are closed.
  - Particles move in circles.
- This means that there is **no net effect** on transport rates through the channel.
- This does however suggest that single profiles of the water column are of limited use.
- The author suggests that measuring for a full Rossby Cycle (80 hours in this case) is necessary before coming to conclusions about layer thicknesses or bottom temperature from CTD data in the region.







**Questions?**