H #7

Question I PO (Andreas Münchow)
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A spatially uniform wind blows along the straight coastline of California. You just finished a hydrographic section across the shelf using a CTD. The density field shown in fig. 1 emerges on your computer screen when your fellow graduate student asks you if it possible that her shipboard ADCP indicates a current of about 2 m/s within 50 m of the surface. She asks you if her instrument is working properly. The chief scientist is sea sick and thus not available because of heavy seas and very strong winds (the kinematic wind stress along the coast τ/ρ is 10^{-4} m^2/s^2, the vertical viscosity A_z of the water is 10^{-1} m^2/s). How do you respond to the request for help? Do you have to wake up the engineer to check the ADCP? Use dynamical arguments (thermal wind, Ekman dynamics) as well as scaling analysis to make a decision base on the following questions:

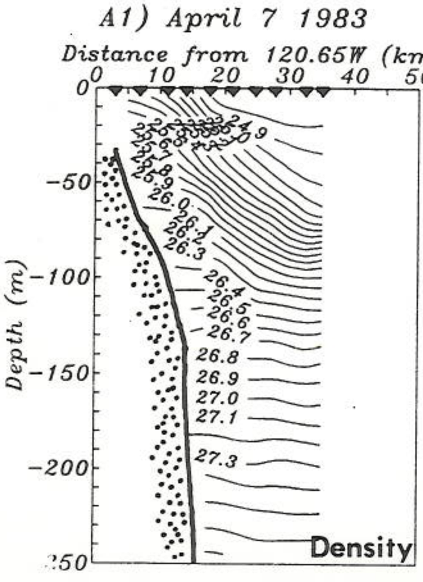
a) Estimate the along-shelf velocity scale of the flow assuming thermal wind holds (the Coriolis parameter f is 10^{-4} s^{-1}). Justify your choice of a reference layer.

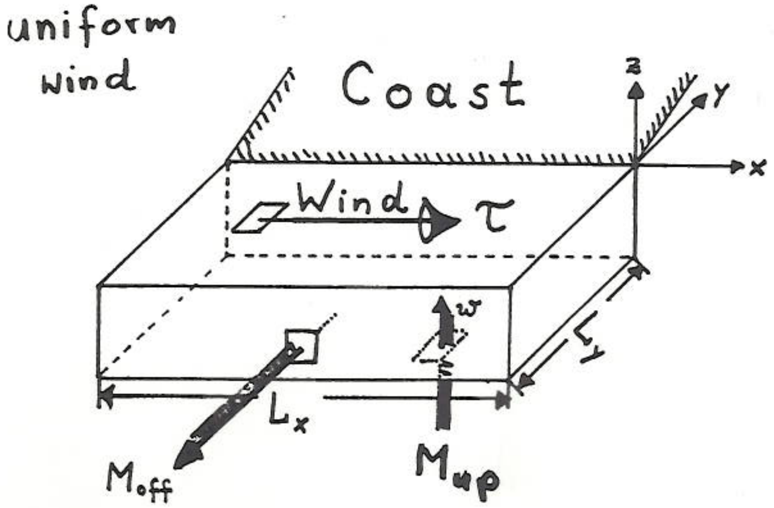
b) Your collegue at the ADCP also has compass or sign problems. Based on your hydrography, advise her on the direction of the flow.

c) Which physical processes neglected in the thermal wind equation could contribute to the flow that could possibly explain the "observed" ADCP velocity magnitude of 2 m/s? Could these processes possibly add up to a total flow of 2 m/s?

d) What are the dynamically relevant length scales L_x, L_y, and L_z (fig. 2) assuming that friction and sloping isopycnals set the the vertical and across-shelf length scales L_z and L_y, respectively. How do the scales compare with those depicted in fig. 1?

e) A biologist finds high nutrient and phytoplankton concentrations within 20 km of the coast. He asks you what the vertical velocities are as he wishes to estimate fluxes into or out of the surface Ekman layer. What is your estimate of a vertical velocity scale if you assume that the across-shelf Ekman flux M_{off} is balanced by a vertical flux M_{up}? Use your length scales L_x, L_y, and L_z from above if you need them.





Homework due Nov. 7, 1996 Physical Oceanography

(A) thermal wind $\frac{\partial v}{\partial z} = -\frac{g}{f\rho}\frac{\partial \rho}{\partial x}$

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after vertical integration

8 $v(z) = v(z_0) - \frac{g}{f}\int_{-z_0}^{z} \frac{1}{\rho}\frac{\partial \rho}{\partial x}\,dz$

8 where z_0 is a refrence layer that from the data I choose at 200m as here the isopycnals are almost flat, i.e., the internal (or baroclinic) pressure gradient vanishes here. Therefore ~~no~~ baroclinic velocity gradients across the ~~slope~~ shelf break are negligible at this depth and I can assume a refrence velocity that does not very with x. I here choose this refrence velocity $v(z_0 = -200m) = 0$

hence

8 $$v(z) = -\frac{g}{f}\int_{-z_0}^{z} \frac{1}{\rho}\frac{\partial \rho}{\partial x}\,dz \approx -\frac{g}{f}\sum_{i=1}^{N}\frac{1}{\rho_i}\frac{\Delta\rho_i}{\Delta x_i}\,\Delta z_i$$

where $\Delta\rho_i$ is the horizontal density difference at vertical location "i"
Δx_i is the horizontal distance over which I estimate $\Delta\rho_i$
Δz_i is a vertical depth increment, and
ρ_i is a mean density at the vertical location "i"

taking $N = 1$, i.e., choosing the crudest possible approximation of the thermal wind integral gives

8 $v(z) \approx -\frac{g}{f\rho}\frac{\Delta\rho}{\Delta x}\Delta z$

≈ -0.5 m/s

$\Delta z \approx 100\,m$ the "height" of the slope $\Delta\rho/\Delta x$
$\Delta x \approx 30\,km$ the "width" of the slope
$\Delta\rho \approx 1.5\,kg/m^3$ density difference of
$\rho \approx 1025\,kg/m^3$ the slope

Thus the velocity difference across the sloping isopycnal is about $\underline{\underline{0.5\ \text{m/s}}}$

(b) The flow is into the page or to the south.
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(c) Geostrophic motions due to a surface slope (or barotropic pressure gradient) are neglected in the thermal wind relation.
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10 Tidal currents (≈ 0.1 m/s); Ekman or frictionally driven currents (≈ 0.2 m/s) due the wind; and nonlinear advective currents could all contribute to the flow. I doubt, though, that all these currents could add up to 2 m/s. Such high flows are generally observed only in narrow straits, strong tidally influenced regions, and western boundary currents such as the Gulf stream. None of these locations or conditions are present here and I'd thus start trouble shooting the ADCP.

(d) 2 L_x, the along-shore length scale does NOT enter the dynamics in any way.

4 L_y could be the lateral scale associated with the sloping isopycnals and a prime candidate is the internal deformation radius which is

$$\sqrt{\frac{\Delta\rho}{\rho}\, g\, H} \Big/ f \approx \sqrt{\frac{1.5}{1025} \cdot 10 \cdot 100} \Big/ 10^{-4} = 12 \text{ km}$$

where H is the vertical scale of horizontal motion; I here associate the sloping pycnocline to cause most of the horizontal motion and thus take $H \approx 100$ m

4 L_z could either be the above depth $H \approx 100$ m or the Ekman layer depth which is about $(2A_z/f)^{1/2} = 45$ m; i.e., the influence of the wind extends about 45 m deep

(e) The Ekman transport offshore is the vertical integral (average) over the Ekman layer depth about $L_z = 45$ m deep times the length L_x of coastline, i.e.,

5 $$M_{off} = L_x \cdot \int_{z=-L_z}^{0} v_{Ekman}\, dz = L_x \cdot \frac{\tau}{\rho f}$$

This offshore transport is balanced by water upwelled over an area $L_x \cdot L_y$, i.e.,

5 $$M_{up} = w \cdot L_x \cdot L_y$$

And thus

5 $$M_{off} = M_{up} \quad \text{or} \quad \cancel{L_x} \cdot \frac{\tau}{\rho f} = w\, L_x \cdot L_y$$

$$\text{and} \quad w = \cancel{L_x} \cdot \frac{\tau}{\rho f} \cdot \frac{1}{\cancel{L_x}\, L_y}$$

$$= \frac{\tau}{\rho f L_y}$$

5 L_x drops out, i.e., it does not contribute to upwelling, using $\tau/\rho = 10^{-4}\ m^2/s^2$, $f = 10^{-4}\ s^{-1}$, and $L_y = 20$ km, I get

$$w \approx 4\ m/day$$

Extra: It thus took about 25 days with upwelling favorable winds to raise the pycnocline (take $\sigma_t = 25.5\ kg/m^3$) from its ambient depth of 100 m about 30 km from the coast to the surface about 10 km from the coast. Once this slope is set up by the winds the ~~deep~~ currents below the Ekman layer (i.e. below 45 m depth) respond geostrophically to the changed density field.