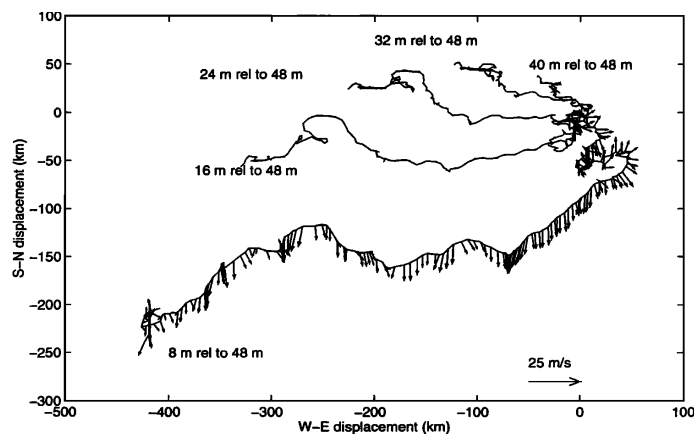


# Final Exam May-22, 2012 (open book, 12-hours)

MAST-806: Geophysical Fluid Dynamics (Spring 2012, Andreas Muenchow)

Section-A [30 pts], 10 quick questions

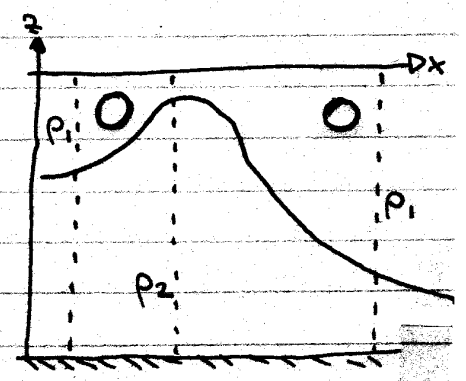
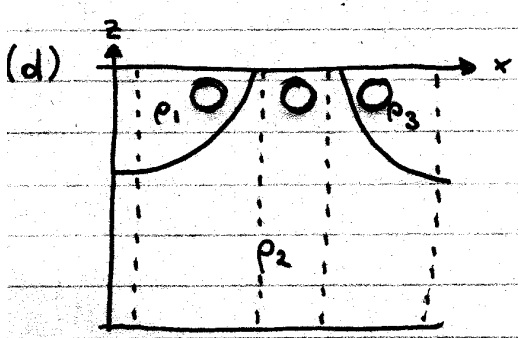
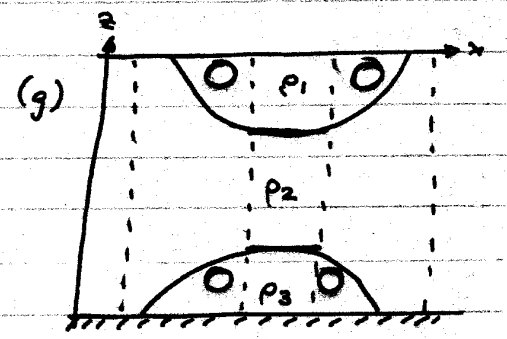
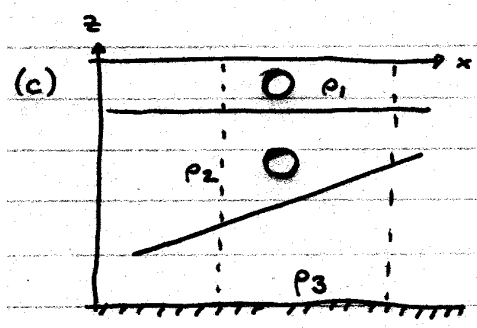
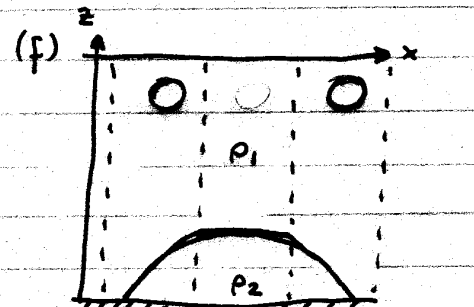
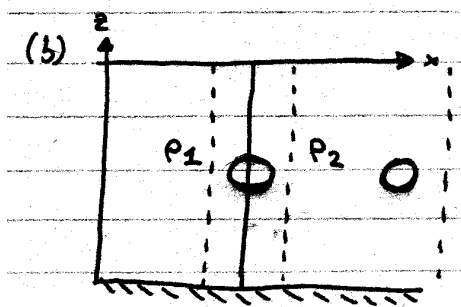
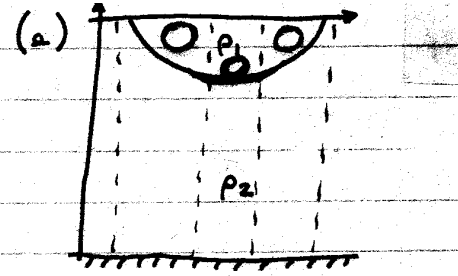
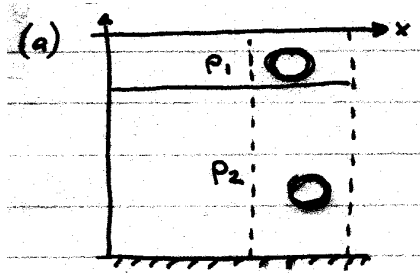
1. For a velocity distribution  $u(y)=U*\sin(2\pi y/\lambda)$ , how would you scale the velocity gradient  $\partial u/\partial y$ ? For  $v=0$ , how would you scale the relative vorticity?
2. Given that the “corrected” gravitational acceleration  $g$  includes a component of the centrifugal acceleration, where do you expect this net or “corrected” gravity  $g=g(\text{latitude})$  to have its maximum and minimum?
3. How would a barotropic, geostrophic flow be impacted by an ice-island 100-m thick? How does your answer change, if the water is 120-m or 5000-m deep?
4. Does the conservation of potential vorticity imply linear or nonlinear dynamics?
5. What is the depth of the ocean that is directly impacted by the wind-stress? Why does this depth differ from the depth impacted by the curl of the wind-stress?
6. What non-dimensional parameter scales the non-linear advective terms in the momentum balance?
7. Does the small parameter in question 6 imply that the dynamics are determined by linear dynamics?
8. If we model the freshwater ( $\rho_1=1020 \text{ kg/m}^3$ ) discharge from Delaware Bay into the ambient ocean ( $\rho_0=1026 \text{ kg/m}^3$ ) at the onset of ebb as a “breaking dam”, what’s the speed at which the front propagates along the coast under the effect of rotation?
9. Can you qualitatively explain these direct ocean current observations that represent “floats” over a 2-month period? Wind vectors are indicated along one of the tracks. Does the data originate from the northern or the southern hemisphere?



10. How could you simulate the  $\beta$ -effect in a rotating tank in a laboratory?

Section-B [10 pts], 1 quick fill-in-the-blanks

Sketch the flow direction based on the thermal wind balance at the indicated circles with  $\rho_1 < \rho_2 < \rho_3$  are densities) relative to no flow at the bottom. Assume a northern hemisphere. Indicate flow into the page as "x" and out of the page as "•" at the locations indicated by open circles.



Section-C [30 pts], Calculation

A spatially uniform winds blows along the straight coastline of California. You just finished a hydrographic section across the shelf and slope. The density field shown in Fig.-1 emerges on your computer screen when your fellow graduate student asks you if it is possible that her shipboard Acoustic Doppler Current Profiler (ADCP) indicates a current of about 2 m/s within 50-m of the surface. She asks you if her instrument is working properly. The chief scientist is sea sick and unavailable because of heavy seas and very strong winds (the kinematic wind stress along the coast is  $10^{-4} \text{ m}^2/\text{s}^2$ , the vertical viscosity of the water is about  $10^{-1} \text{ m}^2/\text{s}$ ). How do you respond to the request for help? Use dynamical arguments as well as scaling analyses to make a decision based on the following questions:

1. Estimate the along-shelf velocity scale of the flow assuming stratified geostrophic dynamics to hold to first order (the Coriolis parameter  $f=10^{-4} \text{ s}^{-1}$ ). Justify your choice of reference layer. [12 pts]
2. Your colleague on the ADCP also has compass problems and is unsure on current direction. Based on your hydrography, advise her on the direction of the flow. [3 pts]
3. Which physical process neglected in the thermal wind equation could contribute to the flow that could possibly explain the “observed” ADCP velocity magnitude of 2 m/s? Could these processes possibly add up to 2 m/s? [6 pts]
4. What are the dynamically relevant length scales  $L_x$ ,  $L_y$ , and  $L_z$  (Fig.-2) assuming that friction and sloping isopycnals set the vertical and across-shelf scales,  $L_z$  and  $L_y$ , respectively. How do these scales compare to those depicted in Fig.-1? [3 pts]
5. A biologist finds high nutrient and phytoplankton concentrations within 20 km of the coast. He asks you what the vertical velocities are as he wishes to estimate flux into or out of the surface Ekman layer. What is your estimate of a vertical velocity if you assume that the across-shelf Ekman flux  $M_{off}$  is balanced by a vertical flux  $M_{up}$ ? Use your length scales  $L_x$ ,  $L_y$ , and  $L_z$  from above, if you need them. [6 pts]

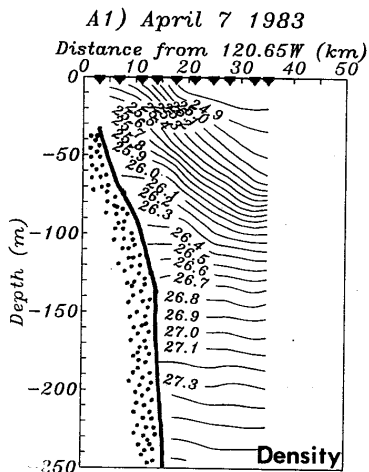


Fig.-1: Density anomaly  $\sigma=\sigma(y,z)$  ( $=\rho-1000 \text{ kg/m}^3$ )

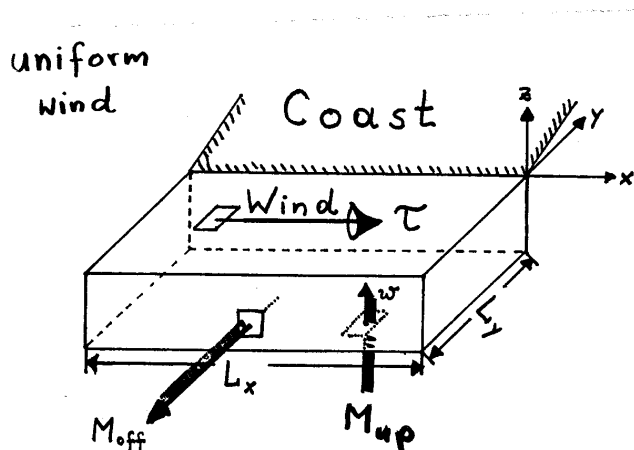


Fig.-2: Sketch of geometry

Section-D [30 pts], open-ended essay

Your friend Tusnelda is beginning her dissertation work on a numerical model of ocean circulation. While Tusnelda is a skilled programmer computers and will use the complete governing equations including nonlinear terms. Her physical insight is weak and has come to regret her frontal attack on such a complicated problem with FORTRAN and fancy compiler design as her only weapons. She has approached you for general guidance to help her anticipate results. [We may even say, she has come to accept John Wheeler's dictum, "Never compute anything until you know the answer."]

Your general task, then, is to write a summary to Tusnelda the modeler describing the physics she can expect her model to produce. Include in your description such properties as spatial and temporal velocity scales she can expect her model to produce.

Her model will include the following simplifications:

- Homogeneous water, steady flow, constant depth  $H=1000$  m
- No lateral friction
- Thin surface and bottom Ekman layers ( $E_v^{1/2} \ll 1$ ) with  $\delta_E = 20$  m
- Beta-plane valid,  $Ro \ll 1$ , i.e., "large scale flow"
- Meridional (north-south) coasts only (at  $x=0$  and  $x=L_x=4000$  km)
- Model domain in southern hemisphere only

Tusnelda will drive the steady flow using only zonal (east-west) wind stress  $\tau^{(x)}(y)$ , i.e., the stress depends only on  $y$  not  $x$ . The stress has a maximum of  $1 \text{ dyne/cm}^2$  that is reached at the southern (open) boundary of the model domain where  $y=0$  and  $f=f_0=-10^{-4} \text{ s}^{-1}$ . The stress vanishes along  $y=L_d$ , the southern hemisphere doldrums, and is such that at the northern (open) boundary where  $y=L_n=3000$  km,  $\tau^{(x)}$  has zero gradient.

1. To begin, assume the interior dynamics for the steady flow obeys Sverdrup balance. For the information given, compute for her the scales for the interior and surface Ekman layer horizontal currents in cm/s. Then, give some justification in terms of the given value for the validity of your assumptions of Sverdrup balance.

2. Make Tusnelda a sketch in the  $(x,y)$  plane of the horizontal distribution of streamfunction  $\Psi$  for the order one flow in the interior. Try to use all the given information about the wind stress, the dynamics, and the boundary conditions. Label features and discuss thoroughly in an accompanying report to him. Given horizontal scales, where possible. Warn him of any possible complications near the coasts where Sverdrup vorticity balance may cease to be valid. Indicate the balance of terms in the potential vorticity equation throughout the domain.

3. Comment on how the interior zonal mass balance is, or is not, achieved inside the model domain.

4. Over the same sketch for part-2 draw a representative vectors showing the horizontal surface Ekman layer flux vector  $M_E$ . (Use a different color for clarity). Comment on possible upwelling circulation at the coastal boundaries.

5. Finally, make a sketch of the flow in the vertical meridional plane along  $x=L_x/2$ . Show representative horizontal and vertical velocities in the interior and both Ekman layers. Indicate the sign of the interior relative vorticity  $\xi_0$  and describe the pressure field.