Mid-term Exam April-10, 2012 (closed book), revised

MAST-806: Geophysical Fluid Dynamics (Spring 2012, Andreas Muenchow)

In class we investigated properties of the quasi-geostrophic Rossy wave that resulted from inviscid, small amplitude motion over linear bottom topography. Here the change in water depth **$h(x)=H_0(x)+\eta(x,y,t)$ is arbitrary**, but the variable bottom slope $\alpha_0(x) \sim H/L$ where H is the vertical scale that h(x) changes in the across-isobath direction x over its scale L. Do some initial investigation of such wave motions as follows

Adopt the linearized, inviscid, shallow water equations, e.g.,

$$\partial_t u - f\,v = -g\,\partial_x \eta \qquad \partial_t v + f\,u = -g\,\partial_y \eta \qquad \partial_t \eta + \partial_x(u\,h) + \partial_y(v\,h) = 0$$

where x and y are the across- and along-isobath directions. Consider wave motions that have frequencies $\sigma<f$ and are **periodic along y with wave number k**. Let V represent the scale for the along-isobath velocity v. Consider f=constant and for simplicity add the restrictions $k\,L \ll 1$ ("long" waves) and $F \equiv f^2 L^2/gH = (L/R)^2 \ll 1$, that is, the across-isobath length scale is small relative to the Rossby radius of deformation $R=(gH)^{1/2}/f$.

1. Show from a scaling argument based on the incompressible continuity equation, that $u \sim k\,L\,V$ thus giving $u/V \sim k\,L \ll 1$. [2 pts.]

2. Perform a scaling analysis of the x-momentum equation to find its simplest form for $k\,L \ll 1$. From this result, **deduce the proper scale for η** in terms of known properties. What is the nature of the across-isobath momentum balance? [6 pts.]

3. By cross-differentiating the simplified x-momentum derived in 2. with the full y-momentum equation (**taking the curl**) and using the continuity equation, find the appropriate vorticity equation. This equation implies the conservation of what property? [7 pts.]

4. Perform a scaling analysis of this vorticity equation to [10 pts]:
   (a) Determine its simplest form.
   (b) Give a verbal description of the dynamic role of each term in (a).
   (c) Obtain an estimate for σ/f. From your results give the frequency an appropriate word description.
   (d) Estimate the along-isobath phase speed in terms of known properties. Does it appear that these waves will be dispersive or not?

5. Perform a scaling analysis of the y-momentum equation. What is the simplest form of this equation? Is geostrophic balance evident? Why or why not? [4 pts.]

6. From your analysis summarize the similarities and differences between these waves and the linear, quasi-geostrophic Rossby waves as you know them. [6 pts.]