Homework-03 Due Thursday April 5, 2012 (prior to class)
MAST-806: Geophysical Fluid Dynamics (Spring 2012, Andreas Muenchow)

1. Show that the scaling of the geostrophic sealevel $\eta^*=(fUL/g)\eta$ where f,U, L, and g are Coriolis parameter (1/s), velocity scale (m/s), length scale (m), and gravitational acceleration (m/s/s) results in (a) an O(1) flow field that is divergence-free and (b) an $O(Ro_T)$ flow field whose divergence gives the O(1) geostrophic sealevel. $Ro_T=1/(f\,T)$ here is the temporal Rossby number, T is the time scale of the motion, and η is the nondimensional sea surface. [The dynamics resulting from this scaling are refered to as quasi-geostrophic dynamics]. [5 pts.]

2. Transform the fully non-linear dimensional continuity equation into non-dimensional form for a water colum $h^*=H\,h$ (h^* and h are dimensional and dimensionless water column heighs, respectively) that contains both a (geostrophically balanced) free surface $\eta^*=(fUL/g)\eta$ and a sloping bottom $H^*=H+\alpha_0 y^* +\eta^*$ where H is a constant scale for the total water depth, α_0 is a bottom slope and y^* is the dimensional distance. Interprete physically the three non-dimensional parameters $\alpha=\alpha_0 L/H$, $Ro=U/fL$, and $L^2/(gH/f)$ as they emerge in the non-dimensional continuity equation from this quasi-geostrophic scaling. [5 pts.]

3. Derive and discuss the dispersion relation for quais-geostrophic topographic Rossby waves for $Ro \sim \beta << Ro_T \sim \alpha << 1 \sim L^2/(gH/f^2)$. Discuss short and long wave limits as well as phase and group propagation characteristics of these vorticity waves in these limits. [15 pts.]