Best to first have the actual vertical velocity distributi as well. start with my data problem.

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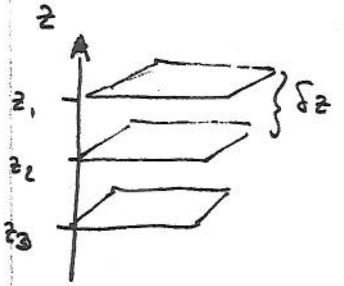
G9.2

Horizontal stress $(\tau^{(x)}, \tau^{(y)})$ at earth's surface force / unit area



stress applied at one level sets fluid in motion thus exerting a stress on the layer δz below

the stress on the layer below is

$$\left(\tau^{(x)} - \delta z \frac{\partial \tau^{(x)}}{\partial z} \,,\; \tau^{y} - \delta z \frac{\partial \tau^{(y)}}{\partial z} \right)$$

but the lower layer exerts a stress $(-\tau^{(x)}, -\tau^{(y)})$ on the upper layer, thus, the net force/unit area will be the difference, that is,

$$\left(\frac{\partial \tau^{(x)}}{\partial z} , \frac{\partial \tau^{(y)}}{\partial z} \right) \cdot \delta z \qquad \Big| \cdot \frac{\delta x \cdot \delta y}{\text{mass}}$$

$$\downarrow \qquad \underset{\text{density}}{\frac{1}{\rho}} \left(\frac{\partial \tau^{(x)}}{\partial z} , \frac{\partial \tau^{(y)}}{\partial z} \right)$$

This is the additional force / mass that tends to accelerate the fluid

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$$E_V = \frac{A_V/\Omega}{H^2} \equiv \left(\frac{\delta_E}{H}\right)^2$$

Linearised momentum equations $Ro \ll 1$, $Ro_T \sim E_V \sim 1$

$u = u(x,y,z,t)$
$v = v(x,y,z,t)$
$\rho = \text{const.}$

$$\frac{\partial u}{\partial t} - fv = \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial \tau^{(x)}}{\partial z}$$

$$\underbrace{\frac{\partial v}{\partial t} + f\cancel{u}}_{\text{Accelerations}} = \underbrace{\underbrace{\frac{1}{\rho}\frac{\partial p}{\partial y}}_{\text{horizontal pressure gradient}} + \underbrace{\frac{1}{\rho}\frac{\partial \tau^{(y)}}{\partial z}}_{\text{vertical stress gradients}}}_{\text{Forcing}}$$

Separate into components ~~and consider separately~~
[linear equations; solutions can always be added up in the end even though they are considered separately]

$$u = u_p + u_E \quad , \quad v = v_p + v_E$$

$$\frac{\partial u_p}{\partial t} + f v_p = -\frac{1}{\rho}\frac{\partial p}{\partial x} \quad , \quad \frac{\partial v_p}{\partial t} - f u_p = -\frac{1}{\rho}\frac{\partial p}{\partial y}$$

Ekman velocities $\left\{ \frac{\partial u_E}{\partial t} + f v_E = +\frac{1}{\rho}\frac{\partial \tau^{(x)}}{\partial z} \quad , \quad \frac{\partial v_E}{\partial t} - f u_E = +\frac{1}{\rho}\frac{\partial \tau^{(y)}}{\partial z} \right.$

The stress $\left(\tau^{(x)}, \tau^{(y)}\right)$ is zero outside the boundary layer, so vertical integration gives

$$\rho\left(\frac{\partial U_E}{\partial t} - f V_E\right) = -\tau_o^{(x)} \quad ; \quad \rho\left(\frac{\partial V_E}{\partial t} + f U_E\right) = -\tau_o^{(y)}$$

sign determines if bottom or surface boundary

"+" boundary is above

"−" boundary is below

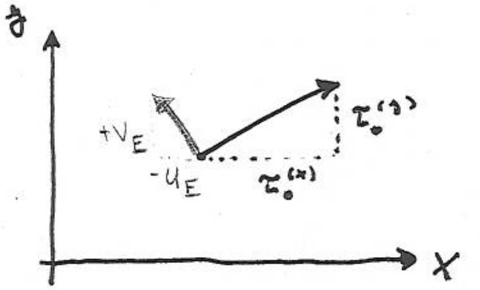
and

$$\left(U_E, V_E\right) = \int \left(u_E, v_E\right) dz = \int \left(u - u_p, v - v_p\right) dz$$

is the volume transport, relative to the pressure gradient flow (Ekman volume transport).

For steady state conditions $\frac{\partial}{\partial t} \cdot = 0$ we have

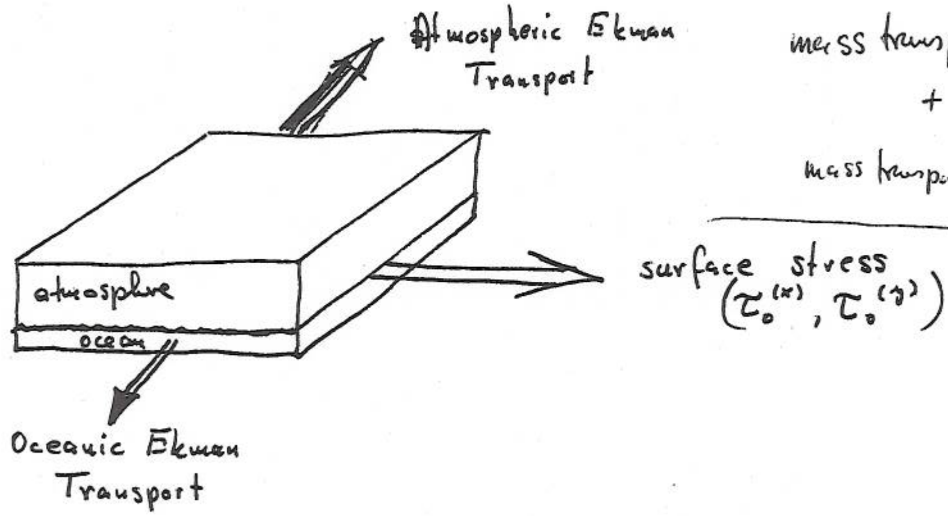
$$-\rho f V_E = -\tau_o^{(x)} \quad \text{and} \quad \rho f U_E = -\tau_o^{(y)}$$



or $U_E \propto -\tau^{(y)}$
$V_E \propto +\tau^{(x)}$

or $\tau^{(x)} \propto V_E$
$\tau^{(y)} \propto -U_E$

Bottom Boundary Layers to the left of stress (northern hemisphere, $f>0$)



(surface) mass transport atmosphere + (surface) mass transport ocean $= 0$

Recall that the right/left angle rule applies only to steady conditions!

$\boxed{G.93}$

Example: Ocean at rest, sudden wind stress $\tau_0^{(x)} = \tau_0$, $\tau_0^{(y)} = 0$
How does the Ekman transport vary?

$$\frac{\partial}{\partial t}(U_E + iV_E) + if(U_E + iV_E) = \frac{1}{\rho}\tau_0$$

(x-mom) + i $\cdot$ (y-momentum) $\quad W = U_E + iV_E$

$$\dot{W} + ifW = \frac{1}{\rho}\tau_0^{(x)}$$

has solution

$$W(t) = -i\left(\frac{\tau_0}{\rho f}\right)\left(1 - e^{-ift}\right)$$

~~at t=0~~

$$) = -i\left(\frac{\tau_0}{\rho f}\right)\left(1 - (\cos ft - i \sin ft)\right)$$

$$u_E = \frac{\tau_o}{\rho f} \sin ft \qquad V_E = \frac{\tau_o}{\rho f}(\cos ft - 1)$$

for small(ft) $\quad \sin ft \approx ft \quad$ and $\quad \cos ft \approx 1$

to $O(ft)$ $\quad u_E \propto ft \qquad V_E \approx 0$

thus for $t \ll f^{-1}$ flow is in the direction of stress

small time ft later:

to $O((ft)^2)$ $\quad \sin ft \approx ft - \frac{(ft)^3}{6} \quad$ and $\quad \cos ft \approx 1 - \frac{(ft)^2}{2}$

$$u_E \propto ft \qquad V_E \propto -(ft)^2$$

The flow begins to veer to the right as $V_E < 0$

Note also that V_E contains both

a time-invariant part and

a time-varying part

The steady state part is the EKMAN TRANSPORT

The time-varying part is an INERTIAL OSCILLATION

Show Gill (1982) Fig. 8.3 and Fig. 9.2

Bowden (1983) Fig. 4.3

Neuman + Pierson (1967) Fig. 8.17

The solution

$$W(t) = -i \frac{\tau_o}{\rho f} \left(1 - e^{-ift}\right)$$

satisfies the initial condition $W(t=0) = 0$

and consists of a steady state part $= -i\, \tau_o / \rho f$ (imaginary!) thus $\perp$ to winds

and an oscillatory component $= +i \dfrac{\tau_o}{\rho f} e^{-ift}$

that can be interpreted as a very long Poincaré wave (inertial oscillation)

If $W(t=t_o) = W_o$ and the wind stess τ_o changes at that time, we get the solution

$$W(t) = \underbrace{-i \frac{\tau_o}{\rho f} \left(1 - e^{-if(t-t_o)}\right)}_{\text{same as before}} + \underbrace{W_o\, e^{-if(t-t_o)}}_{\text{correction to accomodate the new initial condition}}$$

or

$$W(t) = \underbrace{-i \frac{\tau_o}{\rho f}}_{\text{steady state "Ekman layer" response}} + \underbrace{\left(i \frac{\tau_o}{\rho f} + W_o\right)}_{\text{amplitude of inertial oscillations}} e^{-if(t-t_o)}$$

if $W_o = -i \dfrac{\tau_o}{\rho f}$ then there are no i.o.