

Sverdrup Interior

$$\frac{\partial^2 p}{\partial x^2} + 2 \beta_0 H \frac{\partial p}{\partial x} = - \frac{f_0 d}{2} \frac{\partial \bar{v}^{(x)}}{\partial y}$$

## VORTICITY EQUATION

### (1) Sverdrup Interior

$$\frac{\partial p_j}{\partial x} = - \frac{f_0}{\beta_0 H} \frac{\partial \bar{v}^{(x)}}{\partial y}$$

$$p_j(x, y) = - \frac{f_0}{\beta_0 H} \frac{\partial \bar{v}^{(x)}}{\partial y} \cdot x + P_1(y)$$

### (2) Western Boundary

$$\frac{\partial^2 p_b}{\partial x^2} + \alpha \frac{\partial p_b}{\partial x} = 0 \quad \alpha = \frac{2 \beta_0 H}{f_0 d}$$

$$p_b(x, y) = P_2(y) + P_3(y) e^{-\alpha x}$$

Matching conditions

$$\lim_{x_p \rightarrow \infty} p_b(x_p, y) = P_2(y)$$

$$\lim_{x_p \rightarrow 0} p_b(x_p, y) = P_1(y)$$

$P_1(y) = P_2(y)$  will ensure matching solutions

$$p_3(x, y) = P_1(y) - f_0 \frac{\partial \tilde{z}^{(x)}}{\beta_0 H \frac{\partial y}{\partial y}} x \quad \begin{array}{l} \text{no boundary} \\ \text{condition applied} \\ \text{yet} \end{array}$$

$$p_3(x, y) = P_1(y) + P_3(y) e^{-\alpha x} \quad \begin{array}{l} \text{no boundary} \\ \text{condition applied yet} \end{array}$$

Merged Solution valid for both Sverdrup interior and western boundary

$$p(x, y) = P_1(y) - f_0 \frac{\partial \tilde{z}^{(x)}}{\beta_0 H \frac{\partial y}{\partial y}} x + P_3(y) e^{-\alpha x}$$

matching      small within western      small outside  
 boundary layer      western boundary

Now apply boundary conditions :

$$\text{eastern side } x=L, \quad u = -\frac{1}{\rho} \frac{\partial p}{\partial y} = 0 \quad ;$$

$$\text{at } \frac{\partial p}{\partial y}$$

$$\left. \frac{\partial p}{\partial y} = \frac{\partial P_1}{\partial y} - f_0 \frac{\partial^2 \tilde{z}^{(x)}}{\beta_0 H \frac{\partial y^2}{\partial y}} L + \frac{\partial P_3}{\partial y} e^{-\alpha x} \right|_{x=L} = 0$$

$$\left. \frac{\partial P_1}{\partial y} \right|_{x=L} = + \frac{f_0 L}{\beta_0 H} \frac{\partial^2 \tilde{z}^{(x)}}{\partial y^2} \quad \text{or}$$

$$\left. P_1(y) \right|_{x=L} = P_{10} + \frac{f_0 L}{\beta_0 H} \frac{\partial \tilde{z}^{(x)}}{\partial y}$$

western side  $x=0$   $u = -\frac{1}{\rho_0 f} \frac{\partial p}{\partial y} = 0$  :

$$\left. \frac{\partial P_1}{\partial y} \right|_{x=0} + \left. \frac{\partial P_3}{\partial y} e^{-\alpha x} \right|_{x=0} = 0$$

$$\left. \frac{\partial P_1}{\partial y} \right|_{x=0} + \left. \frac{\partial P_3}{\partial y} \right|_{x=0} = 0$$

$$P_1(y) + P_3(y) = P_{30} = \text{constant } @ x=0$$

$$= \Psi(x=0, y) \cdot \rho_0 f$$

where

$$\Psi(x, y) \text{ is a stream function } \frac{\partial \Psi}{\partial y} = -u, \frac{\partial \Psi}{\partial x} = v$$

so that

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = - \frac{\partial^2 \Psi}{\partial y \partial x} + \frac{\partial^2 \Psi}{\partial x \partial y} = 0$$

$$\text{Note that for geostrophy } u = -\frac{1}{\rho_0 f} \frac{\partial p}{\partial y}, v = +\frac{1}{\rho_0 f} \frac{\partial p}{\partial x}$$

$$\Psi = \frac{p}{\rho_0 f} + \text{const}$$

As the constant does not matter, we can set  $P_{30} = \Psi(x=0, y) \cdot \rho_0 f = 0$

Thus

$$\boxed{P_3(y) = -P_1(y)}$$

and thus

$$p(x, y) = P_{10} + \frac{f_0 L_1}{\beta_0 H} \frac{\partial \tilde{\epsilon}^{(x)}}{\partial y} (L_1, -x)$$

$\rightarrow 0 \text{ as } x \rightarrow L_1$

$$- \left( P_{10} + \frac{f_0 L_1}{\beta_0 H} \frac{\partial \tilde{\epsilon}^{(x)}}{\partial y} \right) e^{-\alpha x}$$

$\rightarrow 0 \text{ as } x \rightarrow L_1$

Now evaluate this pressure field at the eastern boundary  $x = L_1$ ,

$$p(x=L_1, y) = P_{10} = p_0 f \Psi(x=L_1, y)$$

but we also want to impose

$$\int_0^{L_1} v dx = 0$$

we have prescribe  
a closed basin

OR

a closed gyre  
bounded by  $\frac{\partial \tilde{\epsilon}^{(x)}}{\partial y} = 0$

in North +  
South

or

$$0 = \int_0^{L_1} \frac{\partial \Psi}{\partial x} dx = \Psi(x=L_1) - \underbrace{\Psi(x=0)}_{\text{we already set this to 0}}$$

on western boundary

$$\therefore \Psi(x=L_1) = 0$$

$$\therefore P_{10} = 0$$

b

$$p(x, y) = \frac{f_0 L_1}{\beta_0 H} \frac{\partial \tilde{\epsilon}^{(x)}}{\partial y} \left[ 1 - \frac{x}{L_1} - e^{-\alpha x} \right]$$